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AND

DISCUSSIONS

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CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
The Design of Rock-Fill Dams. *J. D. Galloway*	Oct., 1937	
Discussion (Author's closure)	Dec., 1937, Jan., Feb., Mar., Apr., May, June, Sept., Nov., 1938	Closed
Multiple-Stage Sewage Sludge Digestion. *A. M. Rawn, A. Perry Banta, and Richard Pomeroy*	Nov., 1937	
Discussion (Authors' closure)	Jan., Mar., Sept., Nov., 1938	Closed
Laboratory Investigation of Flume Traction and Transportation. *Y. L. Chang*	Nov., 1937	
Discussion	Mar., June, Sept., Oct., 1938	Nov., 1938
Stream Pollution in the Ohio River Basin: A Symposium	Jan., 1938	
Discussion	Feb., May, June, 1938	Nov., 1938
Preliminary Design of Suspension Bridges. *Shortridge Hardesty and Harold E. Wessmann*	Jan., 1938	
Discussion	May, June, Sept., Oct., Nov., 1938	Nov., 1938
Relative Flexure Factors for Analyzing Continuous Structures. *Ralph W. Stewart*	Jan., 1938	
Discussion	Mar., May, June, 1938	Nov., 1938
Water-Hammer Pressures in Compound and Branched Pipes. *Robert W. Angus*	Jan., 1938	
Discussion	June, Sept., 1938	Nov., 1938
Engineering Economics and Public Works: A Symposium	Feb., 1938	
Discussion	Mar., May, June, Sept., 1938	Nov., 1938
Flood Routing. *Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder*	Feb., 1938	
Discussion	May, Sept., Oct., 1938	Nov., 1938
Design of Pile Foundations. *C. P. Vetter*	Feb., 1938	
Discussion	May, June, Sept., 1938	Nov., 1938
Progress Report of Committee on Flood-Protection Data	Feb., 1938	
Discussion	June, 1938	Uncertain
Natural Periods of Uniform Cantilever Beams. *Lydik S. Jacobsen*	Mar., 1938	
Discussion	Sept., 1938	Nov., 1938
Pin-Connected Plate Links. *Bruce G. Johnston*	Mar., 1938	
Discussion	Sept., 1938	Nov., 1938
Practical Methods of Re-Zoning Urban Areas. *Hugh E. Young*	Mar., 1938	
Discussion	June, Oct., 1938	Nov., 1938
Progress Report of Committee on Moments in Flat Slabs of Various Types	Mar., 1938	
Discussion	June, 1938	Uncertain
Cost of Energy Generation: Second Symposium on Power Costs	Apr., 1938	
Discussion	Oct., Nov., 1938	Nov., 1938
Economics of Sewage Treatment. *George J. Schroepfer*	Apr., 1938	
Discussion	June, 1938	Nov., 1938
Deoxygenation and Reoxygenation. *C. J. Velz*	Apr., 1938	
Discussion	June, 1938	Nov., 1938
Theory of Silt Transportation. *W. M. Griffith*	May, 1938	
Discussion	Sept., Oct., 1938	Nov., 1938
Water-Softening Plant Design. *W. H. Knox*	May, 1938	
Discussion	Sept., 1938	Nov., 1938
Aerodynamics of the Perisphere and Trylon at World's Fair. *Alexander Klemin, Everett B. Schaefer, and J. G. Beerer, Jr.*	May, 1938	
Discussion	Sept., 1938	Nov., 1938
Observed Effects of Geometric Distortion in Hydraulic Models. *Kenneth B. Nichols*	June, 1938	
Discussion	Sept., Nov., 1938	Dec., 1938
The Three-Point Problem in a Co-Ordinated Field. *R. Robinson Rowe*	June, 1938	
Discussion	Nov., 1938	Dec., 1938
Motor Transportation—A Forward View: A Symposium	June, 1938	
Discussion	Sept., Oct., 1938	Dec., 1938
Relation of Rainfall and Run-Off to Cost of Sewers. *John A. Rousculp*	June, 1938	Dec., 1938
Lateral Earth and Concrete Pressures. *Lazarus White and George Paaswell*	Sept., 1938	
Discussion	Nov., 1938	Dec., 1938
Wind Forces on a Tall Building. *J. Charles Rathbun*	Sept., 1938	
Discussion	Nov., 1938	Dec., 1938
Transportation of Sand and Gravel in a Four-Inch Pipe. *G. W. Howard*	Sept., 1938	Dec., 1938
Principles Applying to Highway Road-Beds. *Ira B. Mullis*	Sept., 1938	Dec., 1938
Settlement Studies of Structures in Egypt. *Gregory P. Tschebotareff*	Oct., 1938	Jan., 1939
Energy Mass Diagrams for Power Studies. *John W. Hackney*	Oct., 1938	Jan., 1939
Beam Constants for Continuous Trusses and Beams. *George L. Epps*	Oct., 1938	Jan., 1939
Solution of Equations in Structural Analysis by Converging Increments. *George H. Dell*	Oct., 1938	Jan., 1939
Reconstruction of a Pier in Boston, Massachusetts, Harbor. *Charles M. Spofford*	Oct., 1938	Jan., 1939
Mechanical Structural Analysis by the Moment Indicator. *Arthur C. Ruge and Ernst O. Schmidt*	Oct., 1938	Jan., 1939
Siphons as Water-Level Regulators. *J. C. Stevens*	Oct., 1938	Jan., 1939

NOTE.—The closing dates herein published, are final except when names of prospective discussers are registered for special extension of time.

CONTENTS FOR NOVEMBER, 1938

PAPERS

PAGE

Great Lakes Transportation.
By M. C. Tyler, M. Am. Soc. C. E. 1731

Transportation Developments in the United States.
By Fred Lavis, M. Am. Soc. C. E. 1751

Analysis of Run-Off Characteristics.
By Otto H. Meyer, Assoc. M. Am. Soc. C. E. 1769

Tests on Built-Up Columns of Structural Aluminum Alloys.
By M. Holt, Jun. Am. Soc. C. E. 1787

Design of Dowels in Transverse Joints of Concrete Pavements.
By Bengt F. Friberg, Assoc. M. Am. Soc. C. E. 1809

Specification and Design of Steel Gusset-Plates.
By T. H. Rust, Assoc. M. Am. Soc. C. E. 1829

State-Wide Surveying Practice in Massachusetts: A Symposium 1847

REPORTS

Land Surveys and Titles: First Progress Report of the Joint Committee of the Real Property Division, American Bar Association and the Surveying and Mapping Division, American Society of Civil Engineers 1879

DISCUSSIONS

The Design of Rock-Fill Dams.
By J. D. Galloway, M. Am. Soc. C. E. 1885

Multiple-Stage Sewage Sludge Digestion.
By A. M. Rawn, M. Am. Soc. C. E., A. Perry Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, Esq. 1895

CONTENTS FOR NOVEMBER, 1938 (*Continued*)

PAGE

Observed Effects of Geometric Distortion in Hydraulic Models.
By E. H. Taylor, Jun. Am. Soc. C. E. 1899

The Three-Point Problem in a Co-Ordinated Field.
By Messrs. Oscar S. Adams, and George D. Whitmore 1902

Wind Forces on a Tall Building.
By Messrs. David Cushman Coyle, and Clyde T. Morris 1907

Preliminary Design of Suspension Bridges.
By Leon S. Moisseiff, M. Am. Soc. C. E. 1910

Cost of Energy Generation: Second Symposium on Power Costs.
By R. H. Parsons, Esq. 1913

Lateral Earth and Concrete Pressures.
By A. E. Cummings, M. Am. Soc. C. E. 1915

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

GREAT LAKES TRANSPORTATION

By M. C. Tyler,[1] M. Am. Soc. C. E.

Synopsis

The Great Lakes and their connecting channels are of great importance in the national economy and the national defense. The steel industry of the United States was largely built up around lake transportation, and the Great Lakes also afford low-cost transportation for grain and coal. This paper is an attempt to outline the history and effects of lake transportation, the works built during a century for the benefit of navigation, and to describe the existing commerce and shipping. The combined Lake and rail movement of bulk commodities is shown to be efficient and economical. The system has been developed gradually through the co-operative effort of the Government, the railroads, industry, States, municipalities, and the owners and operators of vessels. It demonstrates that good engineering practice and sound business judgment, applied to plans laid down only so far ahead as definite benefits can be foreseen, can produce remarkable results.

Historical

Water transportation on the Great Lakes and on the St. Lawrence River and its tributaries has had a tremendous influence on both the political and economic history of the United States and Canada. The first English colonies were established on the Atlantic seaboard in Massachusetts and Virginia at about the same time that Champlain set up his stockade at Quebec in 1608; so the English and the French got practically an even start at conquering the American wilderness. However, water transportation by way of the St. Lawrence River, the Great Lakes, and by the Ohio, Illinois, Fox, and Mississippi Rivers (see Fig. 1), gave the French explorers, missionaries, and traders a great advantage in freedom and rapidity of movement. By 1755, when the English felt strong enough to attempt to push their claims over the Alleghenies and establish themselves in the Ohio Valley, the French, due to water travel and trade, had swung around in front and far ahead of them, and had built missions, forts, and

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Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **January 15, 1939.**

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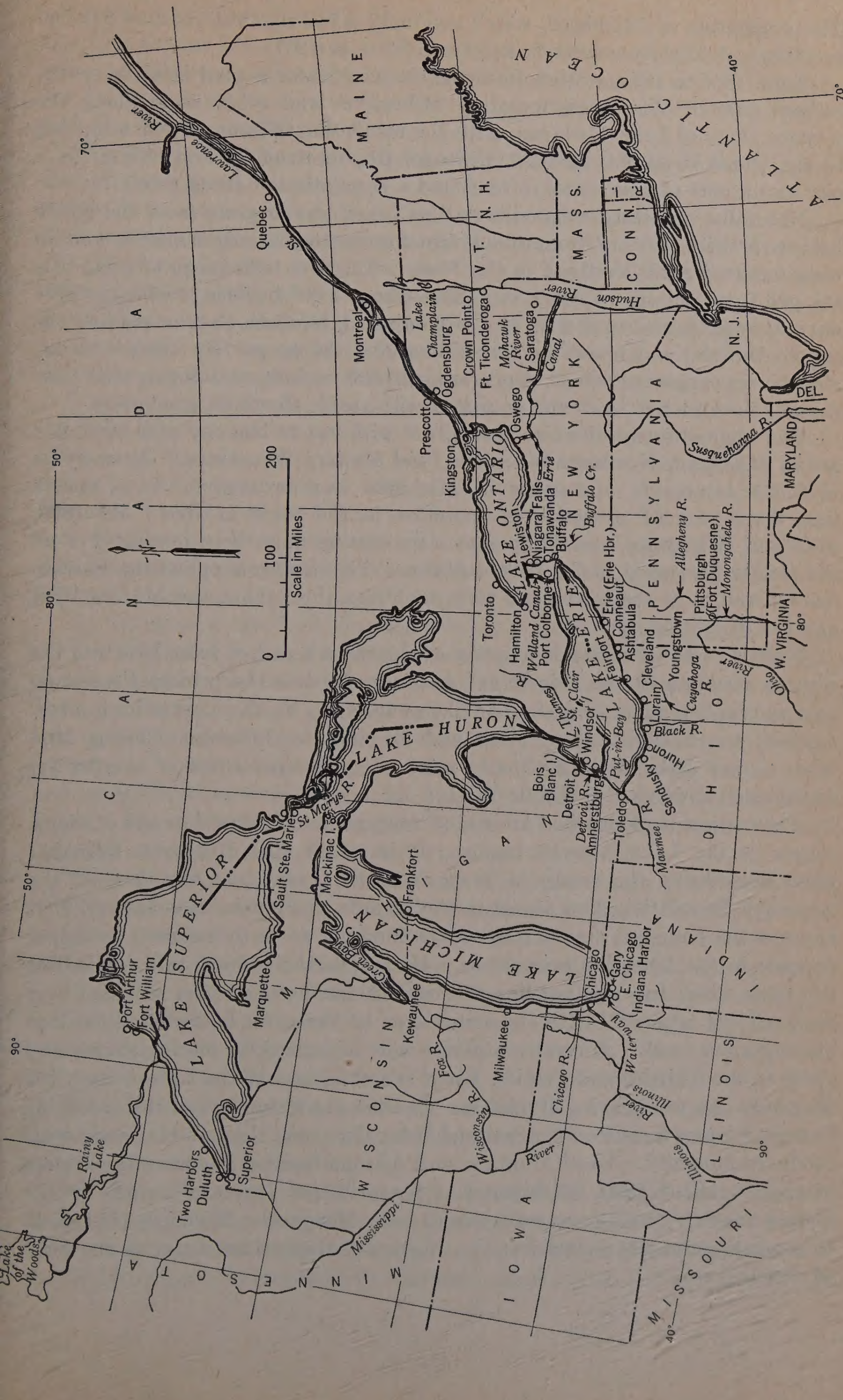
trading posts at Sault Ste. Marie, Mackinac, Green Bay, Niagara, on the Ohio, Illinois, and Mississippi Rivers, and at Ticonderoga and Crown Point, on Lake Champlain. The British had only one post on the Great Lakes, a small one at Oswego on Lake Ontario. When General Edward Braddock made his futile attempt in 1755 to capture Fort Duquesne, a body of French and Wisconsin Indians was in the battle that resulted in his disastrous defeat. The British, moving overland through a wilderness, could make no headway against the French, who were able to concentrate quickly by water at any threatened point. Only when General James Wolfe cut the root of the French water-transportation system by the capture of Quebec in 1759, and Baron Jeffry Amherst, making use of water transportation, proceeded from Oswego with 10 000 men and 1 300 Indians in hundreds of bateaux and captured Montreal, in 1760, did the far-flung French Empire in America collapse, and then New France became British territory.

During the American Revolution, water transportation available to the British from Quebec and Montreal, nearly brought disaster to the Colonial Revolutionary cause. General John Burgoyne reached Saratoga by means of water transport and Sir John Johnson with his Tories and Indians, from Fort Niagara as a water base, raided the settlements in New York and Pennsylvania along the Mohawk and the Susquehanna Rivers. Even after the treaty of peace, water transportation enabled the British to hold the forts on the American side of the Lakes for a number of years in violation of its terms.

Again, in 1813, water transportation on the Lakes played a decisive rôle. The British, with their Indian allies, captured Detroit and Mackinac, and kept General William Henry Harrison's army ineffective on the Maumee River through the command of Lake Erie by a small British squadron which kept Commodore Oliver Hazard Perry's war vessels blockaded in Erie Harbor. It looked as if the United States might lose all of its Northwest Country. For some unknown reason, the British squadron left its station in front of Erie long enough for Commodore Perry, unmolested, to work his vessels across the bar, on which there was about 5 ft of water. The victory over the British squadron at Put-in-Bay followed; the United States regained the advantages of water transportation for movement and supply; and Commodore Perry, using his own and the captured vessels, ferried General Harrison's troops to the Canadian side. Detroit was re-occupied and the British and Indians were defeated in the Battle of the Thames. Had not the control of the Lakes been retaken, it is quite probable that the great iron ore deposits of this country might now be in Canada.

Steamboat navigation began on the Great Lakes in 1816, with a steamer running between Ogdensburg, N. Y., and Lewiston, N. Y., on the Niagara River. The first steamboat on Lake Erie was put in operation in 1818. The opening of the Erie Canal between the Hudson River and Buffalo, N. Y., in 1825 and the Oswego Branch to Lake Ontario in 1828, and, the completion of the first Welland Canal connecting Lake Erie and Lake Ontario around Niagara Falls, in 1829, expedited the settlement of Michigan, Wisconsin, and Northern Illinois. By 1830, a daily boat line was running between Buffalo and Detroit.


C A N A D A
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W. VIRGINIA
NEW YORK
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M I C H I G A N
A T L A N T I C O C E A N
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO
L. St. Clair
Lake Champlain
Lake of the Woods
Rainy Lake
St. Lawrence River
Hudson River
Mohawk River
Erie Canal
Welland Canal
St. Marys R.
Detroit R.
Thames R.
Susquehanna R.
Allegheny R.
Monongahela R.
Ohio River
Cuyahoga R.
Black R.
Huron
Maumee R.
Buffalo Cr.
Fox R.
Wisconsin R.
Chicago R.
Illinois Waterway
Illinois River
Mississippi River
Green Bay
Quebec
Montreal
Prescott
Ogdensburg
Crown Point
Ft. Ticonderoga
Saratoga
Oswego
Kingston
Toronto
Hamilton
Port Colborne
Lewiston
Niagara Falls
Tonawanda
Buffalo
Erie (Erie Hbr.)
Conneaut
Ashtabula
Fairport
Cleveland
Lorain
Youngstown
Pittsburgh (Fort Duquesne)
Sandusky
Put-in-Bay
Toledo
Amherstburg
Detroit
Windsor
Bois Blanc I.
Mackinac I.
Sault Ste. Marie
Frankfort
Marquette
Port Arthur
Fort William
Two Harbors
Duluth
Superior
Kewaunee
Milwaukee
Chicago
Gary
E. Chicago
Indiana Harbor
Scale in Miles
0
100
200
N
40°
50°
70°
80°
90°

FIG. 1.

The population of Michigan, which was only 4 762 in 1810, reached 212 000 in 1840; and Michigan was admitted as a State in 1837.

From 1835 to 1845, settlers from the Eastern States poured into the southeastern part of Wisconsin, founding Milwaukee and other cities along the western shore of Lake Michigan, with the result that Wisconsin was admitted to the Union in 1848. In 1833, Chicago, Ill., received its first charter as a city and a port of entry, and in 1840 had a population of about 4 500.

Naturally, the early commerce on the Lakes was in grain from the newly broken prairies, lumber from the virgin forests to Eastern markets, and in manufactured goods destined to the West. The first bulk cargo of grain was shipped from Chicago to Buffalo in 1839—about 1 600 bushels of wheat. Five days were consumed in getting the cargo aboard, fourteen days in making the run to Buffalo, and seven days in discharging the cargo. In 1840, Chicago shipped six cargoes of wheat, aggregating 10 000 bushels, and during that year a grain elevator was built and equipped with steam elevating machinery.

The original forest of white pine mixed with red or Norway pine stretched across Michigan, Northern Wisconsin, and Eastern Minnesota. As early as 1850 the lumbering operations in Michigan were extensive. It is stated that there were 61 saw-mills in operation in the State in 1854. By 1890, Michigan, Wisconsin, and Minnesota were cutting more than one-third of all the timber harvested in the United States. This timber supplied the Eastern market and built the towns and farm buildings throughout the Middle West and the Northwest.

Buffalo, with its strategic location at the eastern end of Lake Erie and the western terminus of the Erie Canal, naturally became the principal receiving port and milling point for grain, and Tonawanda, N. Y., the market for lumber, destined for Eastern consumption and for export. Likewise, Chicago and other Lower Lake Michigan ports received great quantities of lumber for distribution through the Middle West.

The earliest explorers had known of the presence of large deposits of virgin copper in the Lake Superior region. It is stated that Benjamin Franklin, when negotiating the treaty of peace in Paris, France, at the close of the American Revolution, had access to the records of French explorers and that he drew the boundary line through Lake Superior so as to include the copper deposits in the American possessions. He is reported to have said, "The time will come when drawing that line will be considered the greatest service I ever rendered my country." He did better than he knew, for he included the iron ore deposits as well. Several companies were organized for mining copper and silver in the Thirties and Forties, but it is stated that at the time of the Civil War only two were paying dividends. In 1865, the Calumet and Hecla Mining Company operations were started and from that time thousands of tons were produced annually. Until Montana and Arizona became important producers of copper, about 1880, Michigan was the principal source of supply in the United States. Iron ore was discovered near Marquette, Mich., in 1844, and, on a small scale, was reduced at the mines and shipped by water in the form of excellent iron.

Until 1855 through commerce from Lake Superior was blocked by the rapids in the St. Marys River. Previously, a small quantity of ore was moved from the mines near Marquette by wagon to the Lake, loaded aboard a schooner and taken to Sault Ste. Marie where it was unloaded, carried around the rapids, and reloaded on another vessel.

The State of Michigan, with funds obtained from the sale of lands under a Federal land grant, constructed a canal with locks around the rapids and opened it to commerce in 1855. The first through shipment of iron ore to an Erie port was made in the same year in the brigantine, *Columbia*. The panic of 1857 arrested development temporarily, but the Civil War created a great demand for iron with which to arm, equip, and supply the largest Army and Navy which up to that time the nation had ever been called upon to raise and maintain. Thus, again, Lake transportation became an important factor in military operations and contributed materially to their successful conclusion.

The opening of the St. Marys River to through water transportation made the Lake Superior lumber available to Eastern and Chicago markets. Duluth, Minn., and Superior, Wis., at the head of the Lake, came into being as supply points for lumbermen and, in 1870, were dependent on the lumber business. However, in 1871, wheat began moving from Northern Minnesota by Lake and, rapidly thereafter, with the settlement of North Dakota and Montana, they became the outlet for grain from these States, as well as from Northwestern Minnesota destined for the Eastern markets and for export.

At the close of the Civil War the great era of railroad building throughout the United States required an enormous supply of iron. To meet it, furnaces were built and expanded in the area between Lake Erie and the Monongahela River, where ore by Lake and coal by rail and river could be brought together with a minimum of expenditure of time and money. Thus, the great steel industry in Western Pennsylvania, Ohio, and West Virginia was built by low-cost transportation, combining to the best advantage both rail and water movements. Later, the same principles caused the developments in the steel industry at East Chicago, Indiana Harbor, and Gary, Ind.

Prior to 1884, only the Michigan and Wisconsin ranges were worked, but in that year shipments began from the ranges in Northern Minnesota, which had been discovered about 1875. The Minnesota ores moved and continued to move to Two Harbors, Minn., Duluth, and Superior, for loading into vessels.

With the settlement of the Upper Lake States and the development of industries therein, a demand for coal was created which could be met most economically by combining rail and water movements. Thus, at low costs for transportation, the bituminous coals of Western Pennsylvania, West Virginia, Kentucky, and Ohio have been marketed in ever-increasing quantities throughout the Upper Lake region, farther west in the United States and, to a considerable extent, in Canada.

By 1914, all Europe was at war, and by 1917 the United States was also in the struggle. For two years, the demand of the Allied Powers for steel for arms and munitions was tremendous and then the immense requirements of the United States Government to construct ships and to arm and supply the

Army and Navy were added. The great system of Lake and rail transportation which had been developed in the century since Commodore Perry's victory in 1813, met every requirement of the emergency and had an incalculable weight in the favorable outcome of the war. As long as the iron mines on Lake Superior and the coal mines last, and as long as wars are still a possibility, the transportation system built around water carriage on the Great Lakes will continue to be one of the greatest elements of military and naval strength, better appreciated, perhaps, by the possible enemies of the United States.

THE TRANSPORTATION SYSTEM

A water-transportation system, such as that of the Great Lakes, consists of many elements: The Lakes themselves, their connecting channels both natural and artificial, connecting canals, the harbors and harbor terminals, with their mechanical and other equipment, the railroad facilities serving the harbors and reaching inland, the maintenance of channels and harbors, the vessels in the carrying trade, harbor craft, shipyards, aids to navigation, lighthouses, radio beacons, fog signals, ranges and buoyage, life-saving and distress service, navigation charts and the organizations and men who operate and maintain the vessels, and all the other elements of the system.

It is impossible in a short paper to describe the development and present status of all these elements in detail. However, it may be stated that all of them are in existence and are operated with high efficiency.

CHANNELS AND HARBORS

In their original condition, each of the Great Lakes in itself afforded easy navigation, but connecting channels were either non-existent or inadequate. The first Welland Canal completed by the Canadians in 1829 opened navigation between Lake Ontario and Lake Erie. It had a depth of 8 ft. The first canal around the Falls at the "Soo" was opened in 1855, with a depth of 11.5 ft, connecting Lake Huron and Lake Superior.

There were almost no natural harbors so located along the American shores as to be of commercial value, so the Congress promptly began to authorize the improvement of channels and the construction of harbors by the War Department. For the small sailing vessels and steamers, the rivers and creeks on which the growing Lake communities were located were the only harbor possibilities within the financial abilities of the National Government. In general, the mouths of such waterways were obstructed by bars on which there was as little as 3 or 4 ft of water. Some examples of early harbor improvements are as follows:

(1) A survey of Erie Harbor, where a deep natural bay existed, was made by the Corps of Engineers in 1819 and showed only 6 ft on the bar. By Act of Congress in 1823, a board of officers developed a plan for the improvement of the entrance by dikes and jetties, and work was begun in 1824. By 1828 a minimum depth of 7 ft at all times had been obtained, which was increased to 9 ft in 1830 and to 13 ft in 1868.

(2) Work was begun at the mouth of the Oswego River in 1827.

(3) At Chicago, work at the mouth of the Chicago River was begun in 1833 by the construction of jetties, and a depth of 9 ft was obtained before 1840.

(4) At Lorain, Ohio, parallel jetties were built at the mouth of Black River, work beginning in 1828, and a depth of 10 ft was secured, the original depth having been 3 ft.

(5) Work to improve the entrance to the Cuyahoga River at Cleveland, Ohio, by parallel jetties was begun under the Act of 1825. The original depth of 3 to 4 ft was increased to 6 ft in 1828 and to 10 ft in 1831.

(6) In 1826, work was begun on piers at the mouth of Buffalo Creek, at Buffalo. In practically all these jetties or piers (Items (1) to (6)), timber work filled with stone was utilized for reasons of cost. The Federal Government limited its work to providing entrance channels. The municipalities and vessel operators took care of the channels inside the jetties.

(7) A depth of 12 ft was obtained through the St. Clair Flats in Lake St. Clair in 1855, where the original depth had been from 2 to 6 ft, and St. Marys River was deepened.

(8) By 1869, vessels had so increased in size and number that the need of outer breakwaters to afford anchorages and safety of entrance into the rivers was apparent. Work was begun on the Buffalo breakwaters in 1869, and on those at Chicago in 1871. Timber cribs filled with stone were the standard construction for the earlier breakwaters and most of them, later cut down to low water and capped with concrete, are still in service.

From these modest beginnings, designed to meet the immediate needs of a rapidly growing commerce carried in small vessels, steady progress has been made in deepening and widening connecting channels, in deepening harbors and their entrances, and in giving them more extensive breakwater protection in order to keep pace with the increase in size and draft of the Lake carriers. This has been possible at reasonable costs through the use of the most modern dredging equipment and the substitution of rubble, concrete caissons, and cellular steel shell-pile construction for timber cribs in breakwaters, as the price of timber became excessive.

At the present time (1938), the Lake System consists, in addition to adequate harbors, of down-bound connecting channels 25 ft or more in depth from the head of the Lakes at Duluth-Superior and from Chicago, to Ogdensburg on the St. Lawrence River, with 14-ft channels in the Canadian canals to Montreal, and up-bound channels 21 ft deep. The New York State Barge Canal 12 ft deep from Buffalo and Oswego to the Hudson, being deepened to 14 ft between the Hudson and Oswego, and the Illinois Waterway with a 9-ft depth, are important connecting waterways to the east and south.

Wherever possible, the up-bound and down-bound channels have been separated for safety to vessels. The existing project for the Upper St. Marys River provides for a two-way channel having a minimum width of 1 000 ft, with a depth at low-water datum of 27 ft through Round Island Shoals and

Middle Ground, and 26 ft through Vidal Shoals, widening to 1 200 ft at Brush Point Turn and thence, to Point Louise with a width of 1 000 ft.

The American locks at the "Soo" have the characteristics shown in Table 1. The Canadian Canal at the "Soo" is 1⅛ miles long, 150 ft wide at the top, 142 ft at the bottom, and has a lock 900 ft long and 60 ft wide. Navigation in the canal is limited by the depth over the miter-sills at the locks to boats with drafts of about 17 ft. It was opened to navigation in 1895.

In the Lower St. Marys River, the project provides for a down-bound channel with a depth of 25 ft and a minimum width of 300 ft, and an up-bound channel with a depth of 21 ft and a minimum width of 500 ft. In St. Clair River, the minimum width of the two-way channel is 600 ft, and the depth is 25 ft. In St. Clair Lake the two-way channel is 800 ft wide and 25 ft deep, narrowing to 700 ft at the mouth of the South Channel of St. Clair River.

TABLE 1.—CHARACTERISTICS OF THE AMERICAN LOCKS AT SAULT STE. MARIE, MICHIGAN (DIMENSIONS ARE IN FEET)

Locks (1)	Clear width (2)	Length between miter sills (3)	Lift (4)	Depth on Breast-Wall: Upper (5)	Depth on Breast-Wall: Lower (6)	Type of construction (7)	Year com-pleted (8)
Weitzel	80*	515	21.7	15.8	11.2	Stone masonry	1881
Poe	100	800	21.7	21.8	16.6	Stone masonry	1896
Davis	80	1 350	21.7	24.3	23.1	Concrete	1914
Fourth	80	1 350	21.7	24.3	23.1	Concrete	1919

* 60 ft at the gates.

The two-way channel at the head of Detroit River is 800 ft wide and 25 ft deep. Fighting Island Channel, below Detroit, is 26 ft deep and 800 ft wide; Ballards Reef Channel north of the junction with Livingston Channel is 26 ft deep and 600 ft wide. The Livingston Channel west of Bois Blanc Island for down-bound vessels, is 450 ft wide and 26 ft deep to opposite Bar Point, thence 26 ft deep and 800 ft wide to its junction with Amherstburg Channel, and thence 26 ft deep and 1 200 ft wide to the 26-ft depth contour in Lake Erie, the lower 1 200-ft section being used for both up-bound and down-bound traffic. Amherstburg Channel, east of Bois Blanc Island, for up-bound vessels, is 21 ft deep with a minimum width of 600 ft.

The new Welland Canal, opened in 1932, is 25 miles long, with a bottom width of 200 ft, 310 ft at the water line, and a depth of 25 ft. The canal has seven locks, each having a usable length of 820 ft, a width of 79 ft, and a depth of 30 ft over the sills.

SHIPPING

The great length of the connecting channels in which natural depths were small and the artificial character of the Lake harbors built in comparatively shallow water have made it impracticable to provide depths for Lake navigation equal to that at ocean ports. Storm waves on the Lakes probably never much exceed a height of 20 ft from trough to crest. Most of the freight is handled in bulk. Docks and their unloading equipment, once installed, can be modified only at great expense.

Channel and harbor depths have limited the draft of vessels, dock equipment has limited their beam, and safe handling in restricted waters has limited their length, although the moderate height of storm waves has permitted ratios of length to beam and to depth greater than ocean practice. As a result of all these factors a bulk freighter has been developed with large rudder power, which can carry about as much on 22-ft draft as large ocean vessels can carry on 30 ft, or more. This development, of course, has been gradual with improvements in ship construction and with the increase in depths of channels and harbors.

The first vessels on the Lakes were sailing schooners and brigs; then came wooden-paddle wheel steamers. The *Ontario*, the first American steamer on the Lakes,[2] was built in 1816. It was 110 ft long, with 24 ft beam, $8\frac{1}{2}$ ft depth, a displacement of 237 tons, and was driven by one low-pressure cylinder with a bore of 34 in. and a stroke of 4 ft. It is stated that it took her ten days to make the trip from Lewiston to Ogdensburg. The *Vandalia*, the first propeller boat in the United States, was built in Oswego in 1841. For years, commerce was carried in sailing vessels, towed through the rivers by tugs, in steamers, and in steamers with one or more barges in tow. Hulls went through the evolution in design from wood, to composite wood and iron and steel, and then to steel. The steel whaleback freighter and whaleback barge had their day, but since about 1904 the type of bulk carrier has been well established, the changes being only in their size.

The carrying capacity of Lake bulk freighters on a 19-ft draft ranges from 3 000 to 12 000 short tons (see Items Nos. 1 to 3, Table 2). The extra load per foot of draft, up to 22-ft draft, varies from about 1 000 to 430 net tons. These vessels are usually constructed with the deck-house and high bridges forward, a cargo hold, without bulkheads, and a coal-burning power plant aft. The hatches over the cargo hold are spaced on about 12 ft centers. Trim of the vessels and safety are provided by inside tanks.

Four bulk freighters built during 1937 and 1938 have a carrying capacity on 22-ft draft of 15 440 short tons (Items Nos. 4 to 7, Table 2). They are equipped with geared turbines of a capacity sufficient to obtain speeds of 12 miles per hr loaded and 13.5 miles per hr light. The Canadian steamer, *Lemoyne* (Item No. 8, Table 2), the largest of the Lake fleet, has carried record cargoes in net tons, as follows: Ore, 16 400; bituminous coal, 16 500; anthracite coal, 14 600; stone, 18 100; and, wheat, 17 100.

Self-Unloaders.—In 1928 there were 43 self-unloading vessels on the Great Lakes, with a gross tonnage of 144 730, which were increased to 65 vessels having a gross tonnage of 243 343 in 1937. Many of these vessels are of the usual ore and coal-handling types with cargo hoppers installed in their holds, discharging on to belt conveyors and through a bucket elevator on to a belt conveyor carried on a long boom, which is swung outboard when unloading cargo. Loads from the large vessels can be placed 115 ft from shipside at an elevation of 65 ft above deck level.

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[2] "Freighters of Fortune," Beasley, p. 50.

Motor Ships.—There are 48 motor-operated vessels in bulk and package-freight services on the Lakes, with gross registered tonnages ranging from 1 000 to 8 877, of which 36 are of American registry and 12 Canadian. The two largest, the *Henry Ford II* and *Benson Ford*, are both bulk freighters, of the standard Lake type, propelled by Diesel engines (see Items Nos. 9 and 10, Table 2).

TABLE 2.—TYPICAL VESSELS IN THE GREAT LAKES TRADE

Item No.	Name of vessel (1)	Year built (2)	DIMENSIONS, IN FEET Length (3)	 Breadth (4)	 Draft loaded (5)	 Depth of hold (6)	Gross registered tonnage (7)	Type (8)
1	*George B. Leonard*	1903	380	50	21	23.6	4 037	Lake bulk freighter
2	*Joliet*	1916	504	54	21.5	30.0	6 262	Lake bulk freighter
3	*Thomas W. Lamont*	1930	588	60	22	27.9	7 964	Lake bulk freighter
4	*William A. Irvin*	1937	586	60	22	32.5	8 255	New bulk freighter
5	*Ralph H. Watson*	1937	586	60	22	32.5	8 302	New bulk freighter
6	*Governor Miller*	1937	586	60	22	32.5	8 255	New bulk freighter
7	*John Hulst*	1938	586	60	22	32.5	8 302	New bulk freighter
8	*Lemoyne*	1926	621	70	20	....	10 480	Largest bulk freighter
9	*Henry Ford II*	1924	597	62	17.5	27.8	8 877	Motor vessel
10	*Benson Ford*	1924	596	62	17.5	27.7	8 626	Motor vessel
11	*W. J. Conners*	1901	382	50	19.3	28.0	4 625	Passenger and package vessel
12	*Crescent City*	1897	406	48	20	24.0	4 109	Automobile carrier
13	*James Watt*	1896	405	48	....	23.5	3 807	Automobile carrier
14	*Edgewater*	1931	291	43	12.5	15.7	1 819	Lake and canal freighter
15	*Chester*	1931	291	43	12.5	15.7	1 819	Lake and canal freighter
16	*Badger State*	1912	250	43	10	16.7	1 539	Lake and canal freighter
17	*Panoil*	1920	242	37	18	19.7	1 370	Typical Lakes tanker

Passenger and Package Vessels.—On December 30, 1934, there were 119 vessels of 1 000 gross tons and upward, aggregating 357 231 tons, in the passenger and package freight trade on the Great Lakes. The *W. J. Conners* (Item No. 11, Table 2) is probably typical of this class of vessel. It is powered with an 1 800-hp, quadruple expansion, steam engine, giving it a speed of 13 miles per hr.

Automobile Carriers.—This type of vessel was converted from the bulk freighter. They were converted by the addition of "'tween decks" and elevators for the lowering of cars to the lower decks. A representative vessel of this type is the *Crescent City* (Item No. 12, Table 2). In 1936, a similar conversion was made on the *James Watt* (Item No. 13). This vessel is a pioneer in a rather unique transportation plan—the carrying of loaded truck trailers between Detroit and Cleveland.

Tankers.—Tanker tonnage on the Great Lakes now constitutes one of the important items of traffic. No individual type has been developed in this trade with the exception of the motor tanker that trades on the New York State Barge Canal and the Great Lakes. Several of the tankers in use on the

Great Lakes have seen prior service on the ocean. A typical tanker on the Great Lakes is the *Panoil* (Item No. 17, Table 2), with a speed of 10 miles per hr loaded and 12 miles light.

A large group of Diesel-powered tankers are operated on the Great Lakes by one of the more important oil companies. All these boats are of the New York Barge Canal type, whose general characteristics are a rather excessive breadth, a length of 250 ft, a draft limited to approximately 12 ft, with a speed of approximately 9 miles per hr, and a carrying capacity of approximately 11 000 bbl.

New York State Barge Canal-Boats.—Because of the peculiar physical conditions to be met, a distinct type of boat has been developed for this traffic. All these self-propelled vessels are Diesel-powered with the exception of two boats, the *Edgewater* and the *Chester* (Items Nos. 14 and 15, Table 2). A typical vessel of this type is the *Badger State* (Item No. 16), which draws 10 ft of water, loaded. The power is a Diesel engine of 930 ihp. The cabins are set through the deck with about one-half their height projecting above the main deck. The hold is divided into compartments and closed with the conventional type of hatches.

Items Nos. 14 and 15, Table 2, are more elaborate in construction than other boats in this trade. Power is supplied with two oil-fired water-tube boilers to steam turbines, developing 1 600 shaft hp. The pilot-house is on an elevating mechanism that raises and lowers it. When operating in the canal, the pilot-house is lowered to pass under the bridges, and, in the Lakes, it is raised to give better visibility.

The records show that in 1868, the total number of documented American vessels, exclusive of canal-boats, amounted to 2 543, with a gross tonnage of 454 051, averaging 178 tons per vessel. In 1935, there were 550 American vessels of 1 000 gross tons and more, with a total gross tonnage of 2 575 455, of which 441 were classed as bulk freight carriers, exclusive of tankers, having a gross tonnage of 2 231 883, or 87% of the total tonnage. Canadian vessels of 1 000 gross tons and more numbered 285, having a total gross tonnage of 733 971, of which 221 were classed as bulk freight carriers, aggregating 574 903 gross tons, or 78% of the total gross tonnage.

AIDS TO NAVIGATION

The United States Lake Survey (War Department[3]) publishes for sale and keeps up-to-date general navigation charts of each of the Great Lakes, of Rainy Lake, Lake of the Woods, the New York State Barge Canals, and Lake Champlain, and detailed charts of all harbors and connecting channels and of the St. Lawrence River from Lake Ontario to the International Boundary. It also publishes annually a *Bulletin*, supplementing information given on charts, containing detailed descriptions of shores, outlying shoals, harbors, storm warning, and Coast Guard stations, together with changes in channel conditions and buoyage. The Survey is constantly engaged in sweeping for shoals and wrecks, revising charts, and making observations of Lake levels and river discharges.

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[3] See, Manual of Engineering Practice No. 7, Am. Soc. C. E., Fig. 1, p. 5.

The United States Bureau of Lighthouses (Department of Commerce) has installed and maintains a most modern system of lights, radio beacons, fog signals, ranges and channel buoyage throughout the American waters, harbors, and connecting channels of the Lakes and of the St. Lawrence River. Any changes in lights or buoyage or any other aids are immediately made known to navigators by that Bureau. The Canadian Government performs a similar service in Canadian waters.

The Coast Guard maintains a system of life-saving stations at all important points throughout the Lakes and a fleet of two 165-ft cutters and seventeen patrol boats for the enforcement of Federal laws, for assisting vessels in distress, and for protecting life and property upon the navigable waters of the United States. It also operates, through arrangements with commercial telephone and radio companies and its own radio, a system of communication for vessels in distress.

The United States Weather Bureau operates a system of storm warning stations and forecasts for the Lakes region especially applicable to navigation.

Maintenance of Channels, Canals, and Harbor Works

For the maintenance of channels, canals, and harbor works, and the construction of new works, the War Department maintains five Districts with District Offices in Duluth, Milwaukee, Chicago, Detroit, and Buffalo. The Lake Survey, operated as a District covering the entire Lake region, is also situated at Detroit. All District Engineers are supervised by a Division Engineer, with office in Cleveland, who reports to the Chief of Engineers, U. S. Army, in Washington, D. C. New work of constructing breakwaters or channel dredging is done by contract, and the marine contractors on the Lakes have the most modern equipment for such work.

The War Department maintains four sea-going hopper dredges for the maintenance of harbors, and sufficient other plant to do repair work on breakwaters. Each District Engineer is responsible for the maintenance of project depths in the Government channels in his District and for performing other duties relative to navigation charged by law to the War Department.

Harbor Terminals

There is nothing remarkable about the package freight terminals on the Lakes. The bulk freight terminals, however, are unusual in their excellence of mechanical equipment, capacity, and efficiency. The ore-loading docks are designed with their ore pockets spaced so as to cover the hatches of vessels. The unloading docks for ore are laid out and equipped so as to take ore from vessels and load it directly to railroad car, or into stock-piles. Car dumpers for coal from rail to vessel are of the best design. The bulk terminals in general have been installed by the railroad companies and are supported by adequate and well laid out storage and switching trackage. The efficiency of the Lake system of transportation is due to a combination of economical water haul with highly efficient terminal and railroad operation, making possible the quick turn around of vessels during a seven-month navigation season.

The combined efficiency of terminals and vessels is shown by the records in Table 3. The records of the Lake Carriers' Association show that the average stay of iron-ore carriers at Lower Lake ports has decreased from 36 hr in 1906 to less than 11 hr in 1937. Corresponding figures for ore carriers at loading ports are 22.5 and 4.2 hr.

TABLE 3.—COMBINED EFFICIENCY OF TERMINALS AND VESSELS

Commodity	Unit	(a) LOADING VESSELS		(b) UNLOADING VESSELS	
		Quantity	Time, in hours	Quantity	Time, in hours
(1)	(2)	(3)	(4)	(5)	(6)
Ore	Gross tons	12 508	0.28	12 009	2.42
Coal	Net tons	11 293	3.5	9 366	6.08
Wheat	Bushels	157 565	1	451 382	15

VESSEL OPERATORS AND CREWS

The Lake Carriers' Association was organized at Buffalo in 1892 and was incorporated in 1903. The purpose of the Corporation as set forth in the Charter is:

> "To establish and maintain assembly rooms for the use and general welfare of seamen on the Great Lakes, their connecting and tributary waters; to establish and maintain and procure the establishment and maintenance of aids to navigation; and improve and secure the improvement of channels, docks, wharves, loading and unloading and terminal facilities; to establish and maintain by contract or otherwise such amicable relations between employers and employed as will avoid the public injury that would result from lockouts or strikes in the lake carrying service; to provide for the prompt and amicable adjustment of matters affecting shipping and the interests of vessel owners of the Great Lakes and their connecting and tributary waters, and to lease, rent, purchase or sell such real or personal property as may be necessary or convenient in carrying out the foregoing purposes."

The liberality of view in this statement of purpose and the success of the Association in accomplishing it are remarkable. It has demonstrated how such an Association can be of great service to an industry by studying its problems, improving the conditions of its employees, and presenting facts to the Government agencies with which it has to deal.

The recommendations made by the Association for improvements for the benefit of shipping on the Lakes have consistently been constructive and conservative. At the time of incorporation the membership included passenger vessels, tugs, and sailing ships. On December 31, 1937, the membership included mainly the bulk freight vessels of American registry in the iron ore, coal, grain, and stone trades, as follows:

Bulk freight steamers and motor ships	311
Self-unloading bulk freighters	33
Package freighters	2
Tankers	15
Barges	13
Towing tug	1
Total	375

The total gross registered tonnage of the vessels enrolled in the Association is 2 080 850, or 81% of the total United States gross registry of commercial vessels on the Lakes. The carrying capacity for a single trip at prevailing drafts is approximately 3 000 000 gross tons.

GROWTH IN TONNAGE OF LAKE VESSELS

The number and gross tonnage of vessels enrolled in the Lake Carriers' Association at 10-yr periods, with the change in average tonnage, are as shown in Table 4. In certain cases the Association has expended its own funds for dredging in channels, and the establishment and maintenance of lights, buoys, and lightships when the Government was not in a position to do so.

TABLE 4.—VESSELS ENROLLED IN THE LAKE CARRIERS ASSOCIATION

Year (1)	Number of ships (2)	Gross registered tonnage (3)	Average per ship (4)
1893	612	579 919	946
1905	542	1 411 749	2 605
1915	428	2 012 593	4 700
1925	423	2 223 143	5 255
1935	397	2 185 822	5 500

In 1908, it established a welfare plan for the benefit of sailors, providing for assembly rooms in the principal ports, equipped with current newspapers, magazines, writing materials, and other conveniences, and for the payment of death benefits from accidents incident to employment. The Association maintains an excellent library at the "Soo" and exchanges books with its vessels as they pass through the locks. Through its Committee of Shore Captains, composed of a group of masters with long experience in lake navigation now filling executive posts, its Committee on Aids to Navigation, composed of active masters, and its Fleet Engineers' Committee, made up of former chief engineers now supervising the Engine Departments of the vessels in their respective fleets, the Association is constantly working for safer and better operation and better maintenance and construction.

It has established for its member vessels up-bound and down-bound courses in Lakes Huron, Michigan, and Superior, which also are generally followed by non-Association vessels, thus reducing the hazards of collision in fog.

For twenty years the Association has conducted schools, during the closed season, for the wheelsmen and oilers in its fleet desirous of advancement to officer rating, and for officers desirous of raise in grade. During that period 730 oilers and 544 wheelsmen obtained their original licenses, and more than

1 800 Lake seamen who attended the schools obtained the license for which they were instructed.

The scale of wages paid on Association vessels in 1937 was as follows:

	Per 30-day month		Per 30-day month
Wheelsman	$118.50	Chief cook (on vessels of more than 4 000 tons)	$183.00
Watchman	115.50	Chief cook (on vessels of less than 4 000 tons)	162.00
Deck watch	102.00	Second cook	105.00
Deckhand	87.00	Porter	87.00
Fireman and oiler	115.50		
Coal passer	81.00		

During the World War, under arrangements made with the U. S. Navy Department, 3 503 men were trained aboard the Association's vessels for positions in the Navy and the Shipping Board. In addition, the Association responded to every call from the Emergency Fleet Corporation to provide crews for the ships built or purchased on the Lakes by the Government for ocean service in the war. The offices of the Association furnished 1 934 men for the 112 vessels.

Freight and Passenger Traffic

Now, "the proof of the pudding is in the eating," and the efficiency of any system must be judged by the results. In this sense, the record of the volume of commerce on the Great Lakes and the cost of water transportation speak for themselves.

The freight passing through the canals at the "Soo" (which is the best index of Lake transportation) was only 14 500 tons in 1855; the 500 000-ton mark was passed in 1870; in 1876 more than 1 000 000 tons of traffic utilized this route; and the peak was reached in 1929, when more than 92 000 000 tons of freight passed through the locks.

During the decade, 1926–1935, the average annual commerce of the United States Great Lakes ports was 114 837 300 short tons, which included the highest as well as the lowest tonnages in recent years, with 161 344 361 tons in 1929 and 54 913 140 tons in 1932. After eliminating all known duplications, the net traffic on the Great Lakes in 1936 amounted to 138 373 898 short tons, valued at $1 907 952 846.

Passenger traffic at the United States ports on the Great Lakes consisted largely of ferry passengers during the decade, 1926–1935, and averaged more than 19 000 000 persons, of which 58.5% arrived at, or departed from, Detroit; 20.2% were handled at Lake Erie ports; 6.6%, at Lake Michigan ports; 5.1%, at Port Huron; 5%, at localities on the St. Lawrence River; and the remainder at other Lake ports.

Some idea of the traffic on the connecting waterways of the Lakes can be had from the following data: During the 81 yr from 1855 to 1935, the St. Marys Falls Canal has transited a total of 2 394 472 474 tons of freight, or an average of 29 561 389 tons per year during its existence; the St. Clair Flats Canal, in Michigan, in 1891, transited 22 160 000 tons, which increased to a peak of 99 677 966 tons, in 1929, and, in 1935, amounted to 59 287 907 tons;

the Detroit River with 23 209 619 tons in 1891, reached its high level of 110 719 845 tons in 1929 and, in 1935, shows a total of 75 779 280 tons; the Welland Ship Canal showed a movement of 719 360 tons in 1900, and a peak of 9 280 452 tons in 1929, whereas the 1935 traffic amounted to 8 537 460 tons; the New York State Barge Canal traffic since 1837 (or during 99 yr) amounted to 363 641 280 tons, or an average of 3 673 144 tons per yr. The largest recorded traffic was moved in 1871, when 6 467 888 tons were handled, and the smallest, or 1 159 270 tons, in 1918. Since 1918 the movement has increased to its 1936 volume of 5 014 206 tons; traffic on the St. Lawrence River Canals saw its peak in 1928, when 8 411 538 tons were handled, whereas the traffic since that time has ranged from 5 718 651 to 6 951 064 tons, with a recorded total of 6 873 655 tons handled in 1933, of which 4 879 714 tons originated in Canada and 1 993 941 in the United States.

The Grain Movement.—The territory tributary to the Great Lakes is the most important grain-producing district in the world. It includes most of the surplus grain-producing States of the United States and nearly all the grain territory of Canada. This territory in 1928 produced 35.1% of the world's grain, excluding that of Russia and China, but, in 1935, production had dropped to 28.7 per cent. The sixteen States included within this territory have produced 71% of the wheat grown in the United States since 1919. The enormous quantity of grain flowing eastward is practically all shipped through the four ports of Fort William-Port Arthur, Duluth-Superior, Milwaukee, and Chicago. Shipments from the United States ports are of record from 1868, when a total of 48 585 bushels were shipped, and shipments from Fort William-Port Arthur began in 1884 with 951 bushels. The peak year in grain shipments from Upper Lake ports was in 1928 when more than 572 000 000 bushels were shipped. However, in 1935, shipments from these ports had dropped to 235 000 000 bushels. During the 16-yr period from 1920 to 1935 Fort William-Port Arthur shipped 49.6% of the grain from the four Upper Lake ports; Chicago, 25.1%; Duluth-Superior, 19.4%; and Milwaukee, 5.9 per cent. Wheat constituted 58.3% of the total grain shipments from the four ports; oats, 15.4%; corn, 12.8%; barley, 7.0%; rye, 4.8%; and flaxseed, 1.7 per cent.

Buffalo is the principal receiving port for grain at the Lower Lake ports. It received 200 000 000 bushels in 1928, and more than 104 000 000 bushels in 1935.

Port Colborne, Ont., Canada, is an important transfer point on the Lower Lakes for Canadian grain and received 20 600 000 bushels during 1935. However, since the construction of the new Welland Ship Canal and the construction of transfer elevators at Kingston and Prescott, Ont., Canada, grain en route to Montreal now goes directly to these points for transfer. Kingston received 12 000 000 bushels in 1935 direct from Upper Lake ports in deep-draft vessels, Prescott received 8 000 000 bushels, and Montreal received 51 000 000 bushels in vessels drawing not more than 14 ft of water.

The grain movement on the Great Lakes has changed considerably since 1928. The natural flow of grain has been from west to east, but reduced grain crops during the droughts of 1934, 1935, and 1936 resulted in shipments of grain from the seaboard to the grain-growing sections of the West. During 1928 there were shipments from the seaboard to the Great Lakes *via* the New York State Barge Canal of 230 688 bushels. In 1934, 1935, and 1936, the shipments by this route were 7 452 622, 9 620 572, and 3 671 361 bushels, respectively. In the same years the westbound shipments through the Welland Canal amounted to 11 662 490, 9 214 453, and 9 268 284 bushels, respectively.

In 1937, the westbound movement through the New York State Barge Canal amounted to 3 220 192 bushels and through the Welland Ship Canal to 32 101 679 bushels, a total of 35 321 871 bushels. The receipts at the Upper Lake ports totaled about 30 000 000 bushels and included 21 500 000 bushels of corn from Argentine Republic delivered at Chicago and Duluth-Superior and 1 239 477 bushels of flaxseed from Argentine Republic delivered at Duluth-Superior.

Iron Ore.—More than 1 600 000 000 tons of iron ore have passed down through the St. Marys Falls Canals since 1855. The peak year in the ore trade was 1929 when more than 72 000 000 short tons were handled. In 1936, the ore movement by Lakes was 50 176 000 short tons, of which 47 000 000 short tons passed through the United States locks at the "Soo" from the Superior ranges. The receipts of ore at Lower Lake ports in 1936 were as follows:

Port	Short tons	Port	Short tons
Detroit	1 720 000	Ashtabula	5 017 000
Toledo	1 693 000	Conneaut	7 586 000
Huron	789 000	Erie	1 721 000
Lorain	3 078 000	Buffalo	4 170 000
Cleveland	10 149 000	Port Colborne	77 000
Fairport	1 006 000	Hamilton	521 000

and at Lake Michigan ports there was received:

Port	Short tons
Indiana Harbor	2 527 000
Gary	3 630 000
South Chicago	6 016 000

The movement of ore from Erie ports to interior consuming furnaces is an all-rail movement. The greater bulk of the ore is loaded direct from ship to car. Cleveland, Conneaut, and Ashtabula, Ohio, are the leading Lake ports for shipment to the interior.

Coal.—In 1936, 44 000 000 net tons of bituminous cargo coal were carried, which was 6 000 000 tons more than had ever before been moved on the Lakes in a single season. In addition, the car dumpers put 1 430 000 tons of fuel into the bunkers of vessels which were loading coal. For the first time a single port —Toledo, Ohio—has loaded more than 20 000 000 tons of cargo coal. Sandusky, Ohio, loaded 9 355 000 tons. Shipments from these two ports were almost entirely coal from Southern mines. Shipments of bituminous coal from Lake Erie ports to, or through, the Welland Ship Canal, to a great extent to Hamilton and Toronto, Ont., Canada, reached 3 000 000 tons for the first time.

Distribution of Lake soft coal by districts was as follows:

District	Tons
Lake Superior	11 331 000
Lake Michigan	14 738 000
Lake Huron and Georgian Bay	1 918 000
Lower rivers	6 730 000
Lake Erie	5 057 000
Welland Canal	3 035 000

During 1937, 674 000 tons of anthracite were handled on the Lakes. The value for 1917 was 4 689 000 tons.

Limestone.—In 1936 the Lake movement of limestone amounted to 12 080 000 net tons.

Car-Ferry Traffic.—The first cross-lake service was inaugurated in 1892 between Frankfort, Mich., and Kewaunee, Wis. Since that time this method of transportation has increased until there are three lines operating on Lake Michigan, one on the Straits of Mackinac, three across Detroit River, between Detroit and Windsor, Ont., Canada, one each on Lakes Erie and Ontario, and one across the St. Lawrence River at Ogdensburg. These ten lines operate a total of thirty-five car-ferries, thirty-one of which are in active service. All the car-ferries, except those on Lake Erie, operate during the entire year, trips on Lake Erie being made as traffic demands. During the calendar year 1935, car-ferry traffic on Lake Michigan amounted to more than 3 000 000 tons.

A comparison of the traffic through the Suez, Panama, and St. Marys Falls Canals and of the commerce of three Lake ports is given in Table 5.

TABLE 5.—TRAFFIC OF MAJOR CANALS AND LAKE PORTS

Canal or port	1928		1929		1932		1935	
	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports	Short tons*	Rank among United States ports
(a) TONNAGE THROUGH THE SUEZ, PANAMA, AND ST. MARYS ("SOO"), CANALS								
Suez	31 905 902	..	33 466 014	..	26 049 554	..	29 021 354	..
Panama	33 022 935	..	35 320 308	..	20 334 758	..	27 746 588	..
St. Marys	86 992 997	..	92 622 017	..	20 480 873	..	48 293 308	..
(b) COMMERCE OF LAKE PORTS								
Duluth-Superior	53 204 000	2	60 385 000	2	10 520 000	12	29 185 000	3
Buffalo	21 006 000	5	19 110 000	7	11 146 000	9	13 396 000	10
Toledo	18 539 000	6	19 717 000	6	10 790 000	10	19 631 000	5

* 2 000 lb

Ore.—During 1937 rates on iron ore from the mines at the head of Lake Superior to Lake Erie ports were as follows per gross ton:

Rail to Duluth-Superior and Two Harbors	$0.81
Dumping at dock	0.10
Carriage by vessel to Lake Erie ports and unloading charge to vessel rail	0.80
Vessel rail to car	0.08
Rail to Pittsburgh, Pa.	1.15
Rail to Youngstown, Ohio	0.82

Adding the terminal charges to the water haul gives $0.98 from port to port as against $1.96 for the two rail hauls for ore delivered at Pittsburgh, and $1.63 for ore delivered at Youngstown.

The 1936 vessel rates on coal from Lake Erie ports were as follows:

	Per net ton
To principal ports on Lake Superior	$0.40
To principal docks at Milwaukee	0.50
To principal docks at South Chicago and Indiana Harbor	0.50
To ports on Georgian Bay	0.35

Grain rates vary throughout the season, depending upon the volume of movement and competition for cargoes. The 1936 rates from Duluth-Superior to Buffalo fluctuated between 1.75 cents and 3 cents per bushel. Canadian grain rates from Fort William fluctuated between 3.5 cents and 5 cents to Montreal and between 1.5 cents and 1.75 cents to Buffalo, except for lake loadings when the rate jumped as high as 5 cents.

Cost and Savings

The total expenditures for new work by the War Department on the channels, canals, and harbors of the Great Lakes to June 30, 1937, is $193 573 686.83. It has been estimated by engineers outside the Government service that the savings in freight charges due to water transportation on the Great Lakes are considerably in excess of $200 000 000 per yr.

Conclusion

The existing system of industry and transportation which has been built up around water carriage on the Great Lakes in the last century is the result of the co-operative efforts of the Federal Government, the railroad companies, industries, municipalities, and vessel owners and operators during that entire period. In each decade the facilities provided for navigation and at the terminals have been reasonably adequate for the number and class of vessels in the Lake trade and for the tonnage moved. They have never been overbuilt; nor has money been spent extravagantly far ahead of requirements. As one examines the record, one must be impressed with the fact that the present results have been obtained by the application of good engineering practice and sound business judgment, and by planning for the future only as far as definite benefits could be foreseen.

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PAPERS

TRANSPORTATION DEVELOPMENTS IN THE UNITED STATES

By Fred Lavis,[1] M. Am. Soc. C. E.

Synopsis

A broad review of the problems facing transportation in the United States is offered in this paper, with sharpest emphasis placed upon the problems facing the railroads. Of the five important transportation agencies—railways, highways, waterways, pipe lines, and airways—railways are shown to be the most important.

Special stress is laid on the need for adequate research on a large scale and broad basis into the problems confronting the railroads, both for their proper rehabilitation and necessary progress. The writer considers that the prosperity of the railroads is essential to the prosperity of the country as a whole. The railways are in grave financial difficulties, maintenance is greatly deferred, and progress is not being made. This is due to fundamental defects in governmental regulation rather than loss of traffic due to the general depression. It is believed that there should be better co-ordination of all transport facilities and direct governmental aid or extension of credit should be provided for the railways as it is now being extended to all other forms of transportation.

Importance of the Railways

Twenty-five years ago, when people spoke of transportation in the United States they thought only of railways. At the present time, the railways comprise, as a group, only one of five important transportation agencies—railways, highways, waterways, pipe lines, and airways—all important factors in transportation. Twenty-five years ago the automobile was scarcely out of the crawling stage; the truck, the tractor, and the motor bus were unknown. Aviation was an experiment, pipe lines were not a factor to be reckoned with, and the waterways were little used. Inland waterways had declined as a factor in transportation as the railways advanced, and they had not yet come to the

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Note.—This paper was presented at the Annual Convention, Salt Lake City, Utah, on July 20, 1938, as part of the Symposium on Transportation. Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January 15, 1939.**

[1] Cons. Engr., New York, N. Y.

stage of heavy development in physical structure through the use of Government funds and Federal capital invested in operation.

In spite of the development of these competing forms, however, the railways are still the dominating factor in the field of transportation in the United States; that is, they are dominating in the sense that they move by far the largest volume of freight over the longest distances and, consequently, they are dominating in their relation to general business and industry. With highways so easily visible, and with the dramatization of air travel, the public tends to forget this position of the railways and the fact that they move considerably more than twice as much freight as all the other agencies combined. Exact data as to this distribution are not available, but the following is a reasonably correct estimate of the distribution, between the different kinds of carriers, of the ton-miles of freight handled in 1936:

Description	Ton-miles, in millions	Percentage of total
Steam railways	341 181	67.5
Waterways	92 651	18.3
Motor carriers (highways)	40 227	8.0
Pipe lines (petroleum)	30 710	6.1
Electric railways	704	0.1
Total	505 473	100.0

This list does not include intra-urban transportation. In referring to this classification Mr. Joseph B. Eastman, Member of the Interstate Commerce Commission and formerly Federal Co-Ordinator of Transport, has demonstrated [2] that the ton-mile data do not show the relative effect of highway competition inasmuch as truckage hauls, on the average, are comparatively short. Comparison on a tonnage basis shows a higher percentage of actual tons hauled by trucks, at considerably higher and more profitable rates.

In general, the average cost (to the shipper) of freight hauled on the railroads is slightly less than 1 cent per ton-mile whereas the cost of haulage by truck [3] is from 5 to 7 cents per ton-mile. It is to be realized, of course, that the truck haul is from door to door, whereas the railway haul is the cost of movement on the rails alone.

Another estimate of the relative importance of the various carrier agencies is given by John S. Worley,[4] M. Am. Soc. C. E., as follows:

Description	Gross revenues for 1932	Percentage of total
Steam railways	$3 168 537 000	83.03
Great Lakes	11 733 099	0.31
Pipe lines (oil)	211 788 647	5.55
Inter-city trucks for hire	164 728 000	4.32
Inland waterways	8 572 965	0.22
Electric railways	70 865 505	1.86
Inter-city buses	171 200 000	4.49
Airways	8 600 000	0.22
Total	$3 816 025 216	100.00

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[2] Second Rept. of Co-Ordinator, March, 1934.

[3] "Motor Truck Freight Transportation," U. S. Dept. of Commerce, 1932.

[4] *Engineering News-Record*, June 11, 1936.

These values afford some interesting comparisons although they are for 1932 and the prior list is for 1936. Comparison of the two tabulations serves at least to indicate the very low ton-mile costs to shippers on the waterways as compared to those on either railways or highways. In making these comparisons, however, it must be borne in mind that total costs of transportation are from door-to-door and although, undoubtedly, costs of movement in barges or vessels on waterways are low, they do not necessarily show total costs from origin to destination; neither do they include costs of building and maintaining the waterway itself.

Highway Capacity

It is not possible in the United States, on the basis of present knowledge or expectations, to substitute highway transport for railways for the movement of freight on a large scale. For example, consider the freight movement on the Pittsburgh Division of the Pennsylvania Railroad, which extends between Pittsburgh and Altoona, Pa., a distance of 113.9 miles. On a representative day in 1938, a total of 162 756 net tons of freight was moved in 108 freight trains, containing 4 089 loaded cars, and manned by approximately 575 employees. To move a similar volume of freight by highway, in 5-ton trucks, would require 32 551 vehicles, each manned by at least one employee. If these vehicles were spaced the legal distance of 500 ft apart required by the Pennsylvania State law, they would cover a total of 3 084 miles of highway, or about the distance from Los Angeles, Calif., to Philadelphia, Pa.; and, moving at the rate of 25 miles per hr, would require 123 hr, or more than 5 days, to pass a given point.

Assuming 6 000 motor vehicles per day (that is, vehicles moving in both directions which is a reasonably dense traffic as the normal capacity of a first-class, two-lane highway), it is evident that this freight alone would require about five or six of such highways to handle it. If only half the total traffic were trucks—the remainder being buses, private automobiles, and other vehicles—it would require ten or twelve such highways through this one section of the country to equal the capacity of the one railroad.

Of course, there are several ways of estimating this highway capacity as, for instance, assuming 10-ton, instead of 5-ton, trucks; the fact, however, that the rail traffic cited is fairly representative of an ordinary day must also be considered, but in both cases provision must be made for peak loads. The railroad could handle twice as much traffic on the same trackage; but to handle twice as much in the same period of 24 hr on the highways would require twice as many highways.

The railways are not only important because they now handle by far the largest proportion of the freight but because there now seems no other way in which this traffic can be handled. Furthermore, the World War demonstrated the absolute necessity of railways in time of war.

Present Status of the Art of Transportation

Highway Developments.—The very important subject of highways and motor-vehicle development has been ably treated by Charles F. Kettering [5] and L. I. Hewes,[6] Members, Am. Soc. C. E.; but attention may be called to one phase of the subject, namely, the effect of the speed of motor vehicles on highway design. There are those who claim that all present-day highways are obsolete; that they are out of date as soon as they are built because they are not designed for speeds of 100 miles per hr. They claim that the modern automobile (and possibly, the modern truck) can be, or soon will be, driven safely at this speed; that engineers are lacking in foresight and are hopelessly conservative and behind the times if they do not recognize this need in the design of highways. These writers ignore the fact that only a small proportion of drivers of motor vehicles are capable of driving at speeds in excess of 50 or 60 miles per hr under ordinary every-day conditions.[7] Engineers must take the human element into consideration and not be carried away by too much enthusiasm for modern scientific and mechanical triumphs which, in some cases, may be beyond the ability of the public to use.

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[5] "*Proceedings*, Am. Soc. C. E., June, 1938, p. 1116.

Aviation.—It has been predicted [8] by J. Monroe Johnson, M. Am. Soc. C. E., Assistant Secretary of Commerce, that within a short time 10-hr transcontinental schedules will be established, with great air liners flying in the sub-stratosphere, as well as non-stop sea schedules between New York, London, and Paris; and 40 000 000 persons instead of 400 000 will be using the air regularly for travel. Spencer R. Newman, District Traffic Manager of the United Air Lines, has described [9] the status of aviation in considerable detail.

Realizing that great strides have been made in aviation in recent years, and quite believing that the prediction of Colonel Johnson is not altogether an iridescent dream, the writer doubts that aviation will, for a long time to come, take care of much more than the "cream" of normal increase and the increase in travel which aviation itself creates.

Aviation, highways, and waterways, are handicapped by bad weather conditions to a much greater extent than the railways, and all-year-round, daily, and hourly reliability is still an important factor in the business of transportation.

Waterways.—The subject of navigable inland waterways has been discussed quite fully by R. W. Putnam and S. L. Wonson,[10] Members, Am. Soc. C. E., and C. L. Hale,[11] M. Am. Soc. C. E., who present, quite fully, the arguments for and against these Government subsidized lines of transportation.

The investments of Government capital and the use of Government subsidies under whatever guise they may appear (and in the case of the waterways it is not difficult to recognize them) is a very important consideration in dealing with the transportation problem, but the writer doubts very much if the general public has any idea of the extent of this phase of Government activity, the facts in regard to it, and its effect.

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[6] *Civil Engineering*, September, 1938, p. 585.

[7] "Safety and Speeds as Affecting Highway Design," by F. Lavis, *Proceedings*, Highway Research Board, 1937.

[8] *New York Times*, June 23, 1938.

[9] *Civil Engineering*, October, 1938, p. 679.

[10] "Water Transportation Versus Rail Transportation": A Symposium, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1279.

[11] "Economics of the Ohio River Improvement," *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1527.

Pipe Lines.—Pipe-line operation is a purely commercial development, and is really an extension of the oil-refining and distributing plants of the oil companies. It is part of the system of production, refining, and distribution of their products. The business—the freight—is supplied by the owners of the transportation agency and is part of their business.

Railways.—The development of light-weight Diesel engine trains has been traced by Ralph Budd,[12] M. Am. Soc. C. E., and C. P. Kahler,[13] Assoc. M. Am. Soc. C. E., has described turbo-electric-driven trains.

In the railroad field, at the present time, there are potentialities for improved service, and for the reduction of operating costs, which would almost revolutionize the railway industry and with it create improved business conditions in all industry. Under the conditions obtaining on the railroads to-day, however, these potentialities can bear full fruit only if the human element, financial considerations, and political thought keep step with them. Prosperity and sound business conditions are indispensable for such advances.

PROSPECT OF IMPROVEMENT

Need of Research.—Not long ago advances and developments in engineering and mechanics were products of the thought and research or genius of individuals. Such progress to-day, however, is mostly the result of intensive study by large groups and combinations in laboratories. No longer is this progress the result of the work of the lone man in a garret; it is the co-ordinated effort of large groups of well-paid scientists. They work on definite plans, toward definite objectives, in well-equipped laboratories with, on the one hand, huge testing machines and, on the other, the most delicately made and finely adjusted scientific instruments. Such progress means the expenditure of money. It is supported by groups of large businesses that are prosperous and by this means help maintain their prosperity.

Although a certain amount of research is being conducted by the railways, it is scattered, not well co-ordinated, and is not on a scale that the industry warrants. Some of it is scattered among the colleges and thus is useful, but there is need of a large centrally located and well-equipped laboratory where all these efforts can be supervised.

Advances Depend on Human Element as Well as Science.—Steps are being taken and methods are being put into practice to try to solve at least one of the very difficult problems which the railways have had to face, namely, the loss of passenger business. Attempts to encourage passengers to return to the railways do not begin, however, to touch the fundamental difficulties in which the railways of the United States now find themselves involved. Meritorious though these improvements in passenger service may be, and valuable though they may be in indicating what may be accomplished in the entire field of railway transportation, they should not be allowed to obscure the effects of the much more important factors in advances—the human element, the political element, and the underlying financial difficulties of the railroads to-day.

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[12] *Civil Engineering*, September, 1938, p. 592.

[13] *Loc. cit.*, August, 1938, p. 509.

Recently, the writer experienced the entire gamut of transportation development in one short period of less than two months. He traveled by primitively equipped train, by automobile, and by truck over badly worn dirt roads and no roads, by ox-cart, by mule wagon, motor-boat, horse-back, mule-back, and on foot, over an area about 300 miles in length which required actually five weeks of strenuous endeavor. Then he returned, in the air, over or above the same route, by a regularly scheduled airplane, in $1\frac{1}{2}$ hr. In studying the ways and means of improving the land transport over this route the writer was impressed by the part that had to be played by Man, the human being, by politics, by finance, by commerce, and by economic considerations. All these factors came into play in finding a solution for the problem. The engineering aspects were simple compared to the human, political, and financial aspects.

Advances in transportation, or advances in other fields, mean more than the improvement of existing machines, or the substitution of a new machine for an old one, a Diesel engine or a turbo-generator for the steam locomotive, or the improvement of the steam locomotive itself.

In evaluating the worth of such improvements as increase in speed, stream-lined trains, Diesel engines, electrification, light-weight cars, modernistic design of passenger coaches, and improved freight services, one must have some picture of the railroad situation as a whole and its relation to other transportation developments. If the railroads are "sick," as they are if they are all going into bankruptcy, and they seem to be tending that way, it is of little moment whether or not trains are stream-lined, or whether the decorations in the lounge cars are modernistic, futuristic, or sur-realistic.

Surplus of Facilities.—The outstanding feature of the transportation situation in the United States is the surplus capacity of the combined transportation facilities in relation to the business there is to be done. During the World War, the railways, then virtually the only transportation agency in the country, broke down under the volume of business offered. In the years immediately following, large investments of earnings and capital were made to improve and increase their facilities so that, with the boom period of 1927–1929, they handled the very large volume of business then offered without difficulty.

Government Capital in Highways, Waterways, and Airways

In the meantime, highways were being extended, rebuilt, and repaved, waterways were being improved for navigation, and air lines were beginning to be a factor. The first two were built entirely with Government capital, and the airways were developed under substantial Government subsidies. (Government will be understood to mean not only the Federal Government, but also that of the States, counties, and municipalities.) It is fair to state that, although the highways were, and are being, built with capital furnished by the Government, a considerable part of it came from direct taxes on the vehicles and their operators, and that the subsidies to aviation have been warranted both for mail service and on the score of developing the national defense. It has been announced, however, that the State of Pennsylvania is to provide the sum of $58 000 000 to build a new highway from Pittsburgh to Harrisburg, Pa. Of this, \$32 000 000 is to be a public loan and \$26 000 000 a grant. The loan is expected to be repaid from tolls.

The subsidies to the waterways—that is, the Government investments in waterway improvements—have been justified or excused, in part, on the plea of flood control, or power development, but in the Mississippi Valley they have been made frankly and almost wholly for the improvement of navigation.

The point to be made, is, that the capital for this expansion of highways, waterways, and part of that for airways, was furnished by the Government.

Increases of Transportation Facilities

In his report of March, 1934,[2] as Federal Co-Ordinator of Transport, Mr. Joseph B. Eastman called attention to the fact that in 1920, when the railways had a virtual monopoly of transport, their total investment was approximately 19 billion dollars. Between 1920 and 1932 investments in additional transportation facilities amounted to an additional 20 billion dollars, divided as follows:

Transportation agency	Millions of dollars
Railways, additional	6 300
Pipe lines	424
Streets and highways	12 500
Waterways (U. S. Government)	604
Total	19 828

There were some decreases due to abandonment of electric railways, but the investment in transportation facilities was doubled in that 12-yr period, 1920 to 1932. One-third of it was private capital invested in the railways, and almost two-thirds was Government capital invested in highways.

One of the fundamental aspects of the present-day transportation problem, therefore, is a very greatly increased capacity, a surplus capacity over and above the needs of the country, and the continued and continuing investment of Government capital in additional transportation facilities. It is probable that this surplus capacity would not be so evident or nearly as large if the country had gone ahead at a normal rate of growth during the ten years, 1928–1938, but with depression succeeding depression this increase in capacity becomes one of the major factors in the present situation.

With the almost inevitable further continuance of growth of highways on a large scale, this surplus capacity is not likely to be reduced by population increase or growth of business. This problem, therefore, is likely to continue for some time, aggravated by certain factors that reduce freight traffic permanently.

Passenger Traffic.—Whether or not the passenger traffic can be restored to railroads, or further loss prevented, is difficult to judge. Speaking before the Western Railway Club, at Chicago, Ill., on January 20, 1936, W. W. Colpitts, M. Am. Soc. C. E., expressed great hopes that it might be, by reason of better and faster service. The writer believes that much of the passenger business is lost permanently to the private automobile; but there is a substantial volume that can probably be retained and possibly increased by improvements in service. It is a question, however, if some of this business is not being carried at a loss.

Among some of the adjustments that will be brought about by re-organization and also by voluntary action on the part of the railroads (with the permission of the Government) will be the abandonment of services that do not pay. On July 9, 1938, Federal Judge Carroll C. Hincks over-ruled the Massachusetts Department of Public Utilities and ordered the elimination of eighty-eight passenger stations on the Old Colony Railroad. In his decision he stated that re-organization is impossible, legally and economically, for a railroad that cannot earn its operating expenses. To continue the passenger service of the Old Colony Railroad on the old basis would have jeopardized its capacity to maintain the indispensable freight service in that territory.

Although the changes in service will be inconvenient to some, they were judged to be in the public interest, permitting more satisfactory service for others, and it was reasonable to assume that transportation by highway would be available for those who found themselves without service by rail. Furthermore, according to the decision, the public interest would be served better by a policy protecting indispensable freight and passenger service than by a policy which daily was bringing nearer the destruction of the entire enterprise.

Freight Traffic.—Certain factors that adversely affect the freight business of railways, however, will undoubtedly be permanent. The transportation of coal has not only failed to increase with the increase of population but has actually diminished due to the use of fuel oil and electricity. Both oil and gasoline are now transported by pipe lines and these commodities have the double effect of replacing coal and taking the business from the railways. The decrease in foreign trade, which is not likely to be revived, has decreased the traffic in imports and exports. Much l.c.l. (less car-load lots) freight, especially short-haul freight, is being carried by trucks on the highways, some of these, however, being operated by the railways themselves. The transportation of raw materials has been diminished due to the moving of factories to new locations near sources of supply of the materials they use. Many economies have been introduced by manufacturers to reduce transportation as well as other costs of production.

There is no new territory to develop, the rate of increase in population has decreased, and there is no new immigration. The net remainder, therefore, is a greatly increased transportation capacity and with it certain special factors that reduce the volume of freight to be transported. On "the other side of the ledger" is a certain amount of new business from the automobile industry and the fairly large volume of business that comes to the railways in hauling supplies for the construction of highways and other public works financed by the Government. Some of these benefits will continue unless the nation reaches its limit of borrowing or devaluation, or decides to balance its budget.

One particular phase of freight movement on the railways, which has been developed within recent years, is the need of speed in the movement of certain kinds of goods.[14] Until a comparatively short time ago, large inventories were the rule and orders were placed for quantities to cover seasonal rather than daily or weekly requirements. Then came the present era of small inventories, quick turnover, and so-called "hand-to-mouth" buying.

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[14] *Railway Age*, December 23, 1933, p. 872

To-day, orders are placed by telephone and deliveries from distances as remote as 500 miles may be expected the next day. The result is the development of freight trains traveling at night at speeds of 40 to 50 miles per hr which presents a very different situation from that which obtained 30 to 35 yr ago when grades of 0.2%, 0.3%, and 0.4% were built for heavy trains traveling at 14 to 15 miles per hr, and when shipments were often a week or more on the road.

Is There a Railroad Problem?

The extent of the decline in the business of the railroads is indicated by the data in Table 1. In 1936, Mr. Worley [4] ridiculed the idea that there was then any national transportation problem. He claimed that the railroads were no worse off than other industries and that all were suffering alike from the economic debacle of 1929. Nevertheless, about 75 000 miles of railroads are in the hands of Receivers or Trustees in Bankruptcy to-day (July, 1938), and others seem to be seriously threatened. On July 1, 1938, the Superintendent of Banks of the State of New York announced that more than $3 000 000 000 face value (principal sum) of railroad bonds had been stricken from the list of legal investments for the savings banks of the State. This evidence lends the general appearance of a crisis or, at least, of a gravely serious situation.

TABLE 1.—Decline in Railroad Business, Class 1 Railroads

Item No.	Description	1920	1926	1929	1932	1934	1935
1	Freight originated, in millions of tons	....	....	115.3	....	....	76.3
2	Freight carried, in billions of ton-miles	....	....	447.3	....	....	282.0
3	Gross revenues, in billions of dollars	....	6.3	....	....	....	3.4
4	Net operating income, in millions of dollars	....	....	1 375.0	447.0	....	....
5	Passenger traffic, in billions of passenger miles*	47.0	....	....	....	18.0	....

* *Railway Age*, February 1, 1936.

It must be borne in mind that the railways cannot be compared with ordinary industry, such as a steel mill, a copper mine, a textile plant, or other similar industries, any of which can be shut down in times of low demand. The railroad is compelled to run trains, its need for maintenance goes on more or less regardless of traffic; the service to be rendered, and the rates to be charged are subject to regulation by the Government; and the wages it must pay its employees are (or may be) determined by another governmental agency.

In times of stress the railroads must stop maintenance, stop buying, stop improvements, stop advancing, and this cessation has a very serious reaction on the country at large. During the seven years, 1931 to 1938, the railways are reported[15] to have spent only 49% as much money annually for maintenance as was spent annually in the five years, 1925 to 1929, inclusive. It is estimated that the deferred maintenance at the end of 1937 was approximately a billion dollars.

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[15] *Railway Age*, January 1, 1938, p. 22.

This is partly a matter of judgment but the detailed data cited make it seem reasonable. Keeping maintenance up to date, however, is not advancing, but there can be no advancement at all with any substantial amount of deferred maintenance. Advances cannot be expected in an industry which is in the condition in which the railroads find themselves to-day. If it is true that nothing stands still, then the railroads are going backward, and if they are (and the writer believes they are) it is a serious situation.

Perhaps it is true that the difficulties in which the railroads find themselves are not symptoms exclusively of the railroads but are symptoms of a badly adjusted national economy, the attack on the profit motive, and the drift toward socialistic government ownership of everything. The railroads, however, seem to be bearing the brunt of the attack and, therefore, the problem now is a railroad problem.

Needs of the Railroads

Need of Lower Costs of Operation.—So far as the railways are concerned, economy in the cost of operation means producing ton-miles and passenger miles at the lowest cost consistent with the full movement of traffic, and with the value of the facilities used and service rendered. The last is fundamental. This is particularly so because the service of the future must be much better than that of the past, and must continue to improve.

Labor.—One of the most important factors in this phase of the problem is the cost of labor. According to Thomas F. Woodlock [16] wages to all workers on the railroads now constitute more than 60% of all operating costs and, therefore, they are a most important factor in the situation. Service depends very largely on the human element.

Every one is entirely in favor of adequate compensation for labor—that is, compensation for all those employed from executives to track walkers. However, when special groups of employees banded together for mutual protection (as they have a perfect right to be), use their combined voting strength or group pressure to obtain unfair advantages for this group at the expense of others, or obtain privileges not granted to labor generally, it is not conducive to sound business for the country or the industry.

The very powerful unions of the railway workers have not always worked in the interest of the railroad industry retarding many otherwise possible advancements and, incidentally, injuring their own cause.

There seems to be no good reason why railway employees should be specially singled out and favored over other classes of labor by general legislation as, for instance, in the special Social Security and Pension Acts, applying only to railway employees, passed at the last session of the 75th Congress when this same Congress was too busy to heed the President's plea for substantial relief to the railroads.

There are many angles to the labor question and having been a laborer all his life the writer is entirely in favor of having labor get all the pay the traffic will bear, with reasonable hours of labor and security for old age, but if any or all of these demands are applied unreasonably simply to provide jobs and not to promote efficiency, adjustments will inevitably have to be made, or the jobs will disappear.

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[16] *New York Sun*, January 8, 1936.

That railroad labor has not lagged behind in increases of wages is indicated by the following data cited by Mr. Robert V. Fletcher[17]:

> "The average hourly compensation of railroad employees has increased from 28.3¢ in 1916 to 68.6¢ in 1935, an increase of 142.4%, while the cost of living has increased only 26.8 per cent. The average annual pay increased in this same period from $892 to $1 653, nearly double; but note this, the average number of persons employed dropped from 1 647 097 to 994 371, not much more than half."

Part of this decrease in employment has been due to economies in operation by increasing the number of cars and tons per train, longer runs, elimination of certain terminal and intermediate yards, etc., but these operating economies by the railroads have been more than offset by decreases in the rate level, both freight and passenger, and the general decline in business. The working of the seniority rule has tended to keep an unduly large number of older employees in service and to stifle the ambitions of the younger men, a situation which tends to devitalize the industry.

It is not only that wages to railway employees increased considerably during the twenty years, 1916 to 1935, but these increases have continued through the depression.

At the end of 1937 the bill for wages on the railways was increased by more than $100 000 000 per yr at a time when operating revenues were seriously declining.[18] The result was further curtailment of buying by the railways, an accentuation of the decline in general business activity, and, of course, further decline in business (that is, freight, handled by the railroads) and, a further laying off of large numbers of men. The National Mediation Board which obtained "peace" with the labor unions by granting this increase, did not improve either the general business of the country, the position of the railroads, or, in reality, the economic status of the railroad employees. It is not a healthy condition to have earnings of trainmen increased 16% while average net earnings of the railroads decreased 70 per cent.[19]

The labor question on railroads is a tremendous problem which cannot be ignored. If the railroads of this country are to maintain and improve their position (and there can be no general prosperity without prosperous railroads), labor must do its part in producing ton-miles at the lowest possible cost, and must provide improved service. If this means sacrifices either in number of employees due to more efficient machines, or lower rates of pay, these sacrifices will have to be made; but intelligent co-operation on the part of labor can help produce lower costs without such sacrifices through greater efficiency in operation which, in turn, means better service at lower cost.

The automobile industry is probably the most highly mechanized of any in the country, the value of the product of each man's labor is higher than that of any other industry, and the rates of wages are at the top. This has not been achieved by "made" work but by securing the maximum production from the proper combination of men and machines. Railways will have to achieve something similar to regain and hold prosperity.

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[17] Address to the Academy of Political Science, November 12, 1936, p. 62.

[18] *Railway Age*, January 8, 1938 (quoting from *Washington, D. C. Post*).

[19] Associated Press Dispatch, in New York paper, June 30, 1938.

It must be evident from the foregoing that reduced revenues and increased costs of operation (labor being an important influence in the latter) are two outstanding factors affecting the railroad situation in the United States to-day.

Railroad Capitalization.—There is little use in discussing the present financial structures of the railroads. It has been shown time and time again that the railroads are not over-capitalized. Considerable of the existing capital will probably be lost in the various re-organizations and bankruptcies through which so many of the lines are now passing. The diminishing rate of return on the capital invested in the railroads is shown in Table 2.[20]

TABLE 2.—RAILROAD CAPITALIZATION

Year (1)	Net operating income, in dollars (2)	Annual rate of return (percentage) (3)	Year (1)	Net operating income, in dollars (2)	Annual rate of return (percentage) (3)
1928	373 871 000	5.03	1933	132 595 000	1.93
1929	311 735 000	4.04	1934	118 946 000	1.79
1930	223 692 000	3.10	1935	175 696 000	2.57
1931	121 863 000	1.74	1936	232 782 000	3.53
1932	130 875 000	1.80	1937	119 161 000	1.66

It is very evident that the railways need considerably more capital in order to bring their maintenance up to date, to make advances, and to take advantage of such improvements in the art as are now available in order to produce ton-miles and passenger miles at the lowest possible costs consistent with the improved service which is now being, and will be more and more, demanded in the future.

Present Needs of the Railways.—High-speed, well-equipped, smooth-running, air-conditioned, and probably lighter-weight passenger trains, are needed. Perhaps it is not going too far to state that it would pay to replace most of the present passenger equipment before 1948. Certainly that is true of main-line equipment, except for the comparatively few trains recently put in service. Genuine research is needed, and a mass of facts as to complete costs per unit of transportation of some of the newer forms of equipment and motive power.

The state of the art of building lighter equipment has progressed to the point of planning for an even greater program of replacement of freight equipment than that indicated for passenger equipment. There are possibilities in mechanical refrigeration and other improvements in the mechanical features of freight cars, trucks designed for high speeds, improved draft rigging, air-brakes, etc.; and yet, it is doubtful whether the industry is even making the beginning of a showing into research as to the possibilities of these improvements.

To carry the new equipment and to operate trains at the high speeds to be demanded in the future, considerable attention needs to be paid to the track and signals. No one seems to have been able to improve on the general form of track structure from that of the earliest days; but heavier rails are needed, as are probably welded rails, improved fastenings, still further lengthening of the useful life of ties, more ballast, better drainage, and, of course, elimination of many grade crossings and some revisions of alignment.

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[20] *Railroad Age*, February 12, 1938.

The art of signaling is well advanced and undoubtedly the general installation of some of the newer forms of automatic signals will not only promote safety, permitting increases of speed, but by reason of their efficiency, pay for their original cost.

Railroads have only begun to explore the possibilities of power production. The few examples of electrification, of course, mark important advances, but, in the present state of the art, installations of this kind must be confined to areas of very dense traffic, not only because of cost but from their very nature. The overhead wire system of power transmission seems to be tremendously costly and complicated and perhaps one of the advances in store for railroads is a better means of transmission from the power station to the electric locomotive.

Railroads are experimenting with Diesel engines but these have not yet progressed much beyond the experimental stage for general railroad traction inasmuch as all is not known about their total expense in producing and using power.

The steam locomotive is continually advancing in efficiency and steam as a prime mover is yet the "king" as far as most railroads are concerned. However, there are many more old steam locomotives in service than is desirable, and these should be replaced with modern power.

All these and a myriad other factors can, and should, be developed and put into practice to produce a really efficient railroad transportation machine. Brains, knowledge, and the engineering and scientific ability are available, and nothing is needed but money.

THE MONEY QUESTION

Needed Highway Expenditures.—At the meeting of the Greater New York Safety Council, in New York, N. Y., in 1937, Frank T. Sheets, M. Am. Soc. C. E., read a paper on "The Highway of Tomorrow." He discussed, ably and in great detail, not only the probable future traffic of the highways and their needs but also the economic factors—the costs of delays and the congestion due to lack of sufficient highways of the highest types. According to Mr. Sheets:

> "The blindest can see that tomorrow's highway system must include many classes of facilities. Neglect of one class will cripple the efficiency of all others. The metropolitan citizen must grasp the viewpoint and needs of the small town resident and the viewpoint of the farmer. The latter must see metropolitan highway needs. All must grasp the whole highway picture. Narrowness of vision, selfishness of concept, suspicion, intolerance and misunderstanding are the most formidable barriers to the realization of adequate highway transportation. Planning, engineering and financing are relatively small problems when compared with the public educational work and legislative action which must precede accomplishment."

These sentiments are important and are equally applicable to all transportation problems. They should by no means be confined to those of the highways; and, they certainly apply to railways. After this summing up of the situation the speaker then proceeded to forecast a reasonable estimate of the additional investment which he considered "was required to realize the highway system of tomorrow," and this he fixed at the very modest sum of $57 000 000 000.

Government Funds and Credit.—All this money, of course, is to be provided by Government credit and, again, without discussing the question as to whether the motor vehicle operator or owner will eventually pay his share of it consider what would happen to the railways of the United States if one-tenth this sum (that is, the mere bagatelle of, say, $6 000 000 000) were placed at the disposal of the railways for rehabilitation and progress. Like the owner of the motor vehicle the railroad should then be allowed a fairly wide freedom of operation within the limits of fair dealing with shippers, passengers, and employees.

The Government is extending its credit to the highways and, therefore, incidentally to highway motor transportation and to airways. Assume that this investment will be repaid by taxes or other contributions of the users of these facilities. The fact remains that the Government is financing this progress and looks to the development of business, and to the future users of the facilities, for a return on its investment. In these enterprises the Government takes the place of the bondholders and the stockholders of the railways.

There can be no question but that the Government investment in waterways is a special gift by the entire nation to a small section of the population that can take advantage of these facilities. There is little return on the investment, and there is a further annual expenditure for maintenance. Only recently it was proposed also to invest further millions of public funds in building a system of waterways, permitting the passage of ocean-going vessels from the Great Lakes to the sea, an addition to transportation capacity not needed at all and, from the point of view of many, entirely unsound economically.

Political Aspects.—The Government is a most important factor in this railway situation. If the most important section of the national transportation system is to advance, the Government must do its share. It has undertaken for many years now, to regulate and formulate the policies of the railways in minute detail. It fixes the maximum revenues, it allows or disallows capital issues, it regulates rates and hours of labor, etc. In view of the financial aid it gives to other forms of transportation (which it does not regulate so closely) it should extend some financial aid to the railroads, perhaps only to the extent of the $6 000 000 000, one-tenth of that which highway experts say must be spent on the highways to meet the needs of the motor vehicles. Even quite a little less than this would be very useful.

As early as October, 1932, the President of the United States, speaking at Salt Lake City, Utah, said:

> "There is no reason to disguise the fact that the railways as a whole are in serious difficulty, and when so large a part of the American people have a

direct cash stake in the situation—our job is neither to howl about a calamity nor to gloss over the trouble but patiently and carefully to get to the bottom of the situation—and try to plan for the removal of the basic causes of that trouble."

Many things have been accomplished by the Government since the President made this statement but the basic causes of the "serious difficulty" of the railways have not been removed.

Between 1932 and 1938 the railroad situation, except for a slight flurry in the early part of 1937, got continuously worse and on April 11, 1938, the President in a special message to Congress[21] declared: "Immediate legislation is, I believe, necessary at this session in order to prevent serious financial and operating difficulties between now and the convening of the next Congress."

However, in a recent "Fireside Chat" to the people over the radio, the President complained to his audience that: "The Congress also failed to meet my suggestion that it take the far-reaching steps necessary to put the railroads of the country back on their feet."

In spite of the request of the President for immediate relief and in spite of the appropriation by this same Congress of between $4 000 000 000 and $5 000 000 000 for various public works, which undoubtedly will include further construction of highways, waterways, and airways, nothing was done about the relief to the railroads which the President said was so greatly needed. However, in June, 1938, this same Congress did find time to pass special legislation for the railway employees.

Suggested Remedies

Consolidation and Co-Ordination.—The questions of consolidation, co-ordination, and competition are always foremost in any discussion of the railroad situation. Personally, the writer is opposed to consolidation when carried to the extent of building up such large organizations that they lose the sense of personal management. There are undoubtedly cases where further consolidation may be desirable, but these should be studied as individual cases rather than on the basis of some arbitrary plan for the grouping of all the railroads of the country, or of some section of the country.

Co-ordination is being developed between the railways and the highways, waterways, and airways. The experience of the Federal Co-Ordinator of Transport, however, has shown that this is no one-man job; probably it mus be developed along the lines of natural growth—in other words, by the American method rather than that of governmental paternalism.

Competition.—Competition has two aspects. In one sense competition is waste, inasmuch as it involves the use of duplicated facilities, but, in the case of the railroads, it is not as injurious or wasteful as it may seem on the surface. The second aspect, which compensates for any disadvantages of the first, is that competition is a healthy state for an industry such as railroad transportation, which depends so much on the human element which, in turn, competition tends to keep alert. There is certainly competition in commercial highway transportation and in commercial aviation. It would be wise to continue reasonable competition in the railway field.

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[21] H. R. Doc. No. 583, 75th Cong., 3d Session, April 11, 1938.

In a review of the railway situation presented to the Alumni Association of the Harvard Business School on June 17, 1938, Mr. Eastman referred to the various "schools of thought" in suggested remedies for the present situation of the railways which he said were sick, financially speaking, and are the subject of general and grave concern.

He referred to the school of thought that believes that the solution of the problem is to be found in putting the capitalization of the companies "through the wringer"; another school believes in raising rates and fares; and another believes that the entire trouble is that of wages and working conditions.

He emphasizes the fallacies of the first two and says of the third that although drastic changes in the wage scale are not feasible, he does believe that adjustments are necessary. He finally expresses the belief that, in view of the position it has taken and built up in the past, the Government must be an important factor in the necessary rehabilitation and building up of the railways.

Mr. Eastman states that, inasmuch as the Supreme Court has ruled that transportation is a "function of the State," essential to the welfare and well-being of the entire community, the Government must play a more active part in the field of planning, promotion, and prevention.

The Solution

Now, what is the answer to all the foregoing questions? They all revolve about one question—advances in transportation. The railways are clearly the largest factor, and it is evident that they must be maintained in that position if the country is to continue to prosper and if general industry is to be served. Nevertheless, the railroad industry, on the whole, is going backward instead of advancing and this is not due, except in degree, to the present depression. It will not be really cured by such advance in prosperity as can now be visualized.

The labor situation needs adjustment. If the railways must compete with other industries for business they should be allowed to compete and "sink or swim" by their own efforts as other businesses must, and this need not involve indiscriminate "throat cutting" on rates.

The writer has avoided the mention of taxes because it is too big a subject for the scope of the paper; but taxes are growing ever larger. It seems as if the tax structure might well be adjusted; but there are many opinions about that and, in any event, it will add little to the present thesis to do more than mention it as a factor in the situation.

Government to Furnish Capital and Credit to Railways.—In the matter of capital needs and credit, the railways should be placed on an equality with highways, airways, and waterways, and they should have the full advantage of Government credit and the supply of Government funds for their necessary rehabilitation and progress. If this is done the Government should be in the same position as to railways properties that it is to the highways; that is, a stockholder looking for a return on his investment to the earnings of the property, after existing funded indebtedness has been taken care of.

Investments that are not legal for savings banks are usually not legal for insurance companies and insurance companies are large holders of railway securities. This, then, is not a question of a few powerful and affluent bankers, or wealthy families, but of almost every holder of a savings account or an insurance policy, and that means nearly all the ordinary, every-day, plain careful citizens of the country.

The loaning of Government credit and capital to these people is at least as much justified as is the loaning of Government money to the manufacturers of automobiles, gasoline, and oil, through the construction of highways on which the automobiles may run and the gasoline and oil may be used.

There seems to be ample precedent for the extension of Government credit and the making of Government loans to the railways, and if the railroads are to avoid the Government ownership and operation to which the President says he is opposed, there is little else that can be done.

There seems to be no valid argument against maintaining the present financial structures intact and subject to the orderly processes of present re-organization proceedings. Railroads have been "shaken down," their weaknesses have been exposed, some non-useful and non-profitable services are being abandoned. The President of the United States has spoken very strongly against Government ownership and operation. Viewing the transportation system of the United States as a whole, therefore, it seems reasonable that, in the matter of credit and capital, the Government should put the railways on a par with the highways, waterways, and airways, and then let them run their own business with a minimum of regulation and interference

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

ANALYSIS OF RUN-OFF CHARACTERISTICS

By Otto H. Meyer,[1] Assoc. M. Am. Soc. C. E.

Synopsis

An analysis of the characteristics of run-off hydrographs is outlined in this paper, which proposes a revision of current methods for the determination of stream flow from rainfall. It is shown that a continuous rain of constant intensity produces a hydrograph of characteristic shape for any drainage area, and that this shape and the length of time until the run-off becomes constant (time of concentration) are functions of the shape and size of the drainage area. A method of constructing a hydrograph resulting from rain of duration less than the time of concentration is proposed. Based on this analysis, methods have been devised for determining hydrographs for areas lacking in run-off records, from the records of other areas similar in characteristics but different in size, and for the construction from rainfall data of hydrographs for storms or series of storms of varying intensity.

Unit Hydrographs and Distribution Graphs

Hydrographs resulting from isolated storms have long been known to have characteristic shapes. LeRoy K. Sherman, M. Am. Soc. C. E., found that for any drainage area all hydrographs resulting from isolated storms of a certain duration had approximately the same length of base, and that when reduced proportionally to correspond to the same volume of run-off they had very similar shapes.[2] He proposed the "unit hydrograph," or the hydrograph resulting from a 1-in. run-off from precipitation occurring in a unit of time (usually 24 hr). When its ordinates are multiplied by the inches of run-off from any storm of unit duration, this unit hydrograph gives the hydrograph resulting from that storm.

Merrill M. Bernard, M. Am. Soc. C. E., proposed a "distribution graph," in shape similar to the unit hydrograph, but representing, by its ordinates, percentages of the total run-off from a storm, or the proportional distribution of

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January 15, 1939**.

[1] Asst. Engr., U. S. Engr. Dept., Sacramento, Calif.

[2] *Engineering News-Record*, April 7, 1932.

run-off with respect to time.[3] The distribution graph is derived by dividing the ordinates of the unit hydrograph by the total volume of run-off, as indicated by the area under the graph. A refinement of the method of using a single actual hydrograph to determine the distribution graph is to compute distribution graphs for a number of storms, plot them together, fitted as well as possible to each other, and draw a mean graph, thus producing the most probably representative distribution graph.[4]

A method of constructing the composite hydrograph resulting from a series of rains, or rains lasting through several units of time, has been developed and is in general use.[4] The procedure is to compute the hydrograph resulting from the rain in each unit of time, and to add the resulting hydrographs directly. The composite hydrograph thus constructed will fit the actual observed hydrograph fairly closely.

At best, stream-flow records are meager, and hydrographs are frequently required for points at which no gages have been established. Under such circumstances it is required to determine an approximate distribution graph by transposition from another drainage area or from another part of the same area. Mr. Sherman has proposed a method of determining a unit hydrograph for any size of drainage area from the known hydrograph of a drainage area of similar characteristics.[5] This method (based on the principle that all dimensions of similar areas vary as the square roots of the areas) is, in brief, to increase or reduce the ordinates and abscissas of the unit hydrograph in proportion to the square roots of the respective drainage areas. This similarity of drainage areas applies particularly to slope, nature of the topography, extent of vegetation, and distribution of tributaries. For the relation to hold requires that the mean velocity of the water remain approximately constant through the length of the water-shed. However, the velocity of flowing water, which determines the length of the unit hydrograph, depends not only on slope (which the transposition leaves unchanged although small drainage areas have steeper slopes than large ones), but also on hydraulic radius and stream-bed roughness, which vary with the size of the drainage area. The velocity, therefore, should not be assumed to be constant. It is thus apparent that reliance cannot be placed on the foregoing method. An attempt was made by the writer, using this method, to determine hydrographs at successive points on the Suncook River in New Hampshire, with indifferent success. It is true that each of the hydrographs constructed was, in itself, a reasonably good approximation, but comparisons between them failed to give the information required in this case, which involved the routing of natural and reservoir-modified flood hydrographs. Obviously, a real necessity for a more refined method exists.

Defects similar to the foregoing exist also in the following formulas proposed by the Committee on Floods,[6] of the Boston Society of Civil Engineers: $T \propto \sqrt{M}$; and,

$$Q_p = C_f \sqrt{M}\, I \qquad (1)$$

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[3] "An Approach to Determinate Stream Flow," by Merrill M. Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

[4] "Unit Hydrograph Analysis of Surface Runoff," U. S. Geological Survey, *Water Supply Paper 772*.

[5] *Transactions*, Am. Geophysical Union, 13th Annual Meeting, 1932, p. 332.

[6] *Journal*, Boston Soc. of Civ. Engrs., September, 1932, p. 388.

in which M is the drainage area, in square miles; T is the total flood period, in days; Q is the peak discharge, in cubic feet per second; C_f is a flood coefficient, and I is total rainfall, in inches. These relations, involving, respectively, the abscissas and ordinates of the hydrograph, are in effect the same as those assumed by Mr. Sherman. According to the report of the Committee, these relationships, which are only approximately borne out by actual observations, are based on the assumption that the drainage area involved is rectangular and of uniform slope, like a shingle on a roof, and without any well-defined channels.

Analysis of the Hydrograph

It is well known that run-off is not a function of rainfall alone, but involves such factors as infiltration, evaporation, and surface pondage. The relationship between rainfall and the actual flow into streams is still the subject of much discussion, but probably it will be generally conceded that after a short time lag the run-off will be approximately proportional to the rainfall. Rainfall intensities vary considerably, and uniform, continuous rainfall is actually an unknown phenomenon. For these reasons, and because fluctuations in the intensity of a rain of considerable duration have only a minor influence on the shape of the hydrograph, it is assumed permissible to treat the run-off as a constant percentage of a uniform rain. Rainfall rates and percentages of run-off are assumed constant, therefore, unless otherwise specifically stated. It is also assumed that ground-water flow is separable from flood flow, and, consequently, it is assumed to be separated and is not mentioned in the analysis of flood flows. Uniform distribution of rainfall over the entire drainage area is also assumed, although it is understood that this condition is never realized, and is only approached on small water-sheds; in actual cases the hydrograph is not expected to vary greatly in shape from one produced by rain of uniform distribution.

Assuming a uniform, continuous rain, the run-off passing any point on the stream will increase as water from more distant parts of the drainage area comes to the point of measurement, reaching a maximum and becoming constant when water from the farthest extremity of the drainage area arrives. This time, until water from the most distant part of the drainage area reaches the gage, is known as the time of concentration.

If, after the run-off has become constant, the rainfall ceases, all water in surface detention soon runs off into the stream beds, soaks into the ground, or evaporates. As the streams then have no source of replenishment other than ground-water flow, which has been eliminated from consideration, all flow thereafter must come from depletion of the water in channel or valley storage. The run-off then follows a descending curve which W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E.,[7] found to follow the general equation:

$$Q_d = \frac{Q_m}{K^t - t_l} \qquad \text{(2)}$$

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[7] "Relation Between Rainfall and Run-Off from Small Urban Areas," *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (2), p. 151.

in which Q_d is the discharge, in cubic feet per second, on the descending limb of the hydrograph; Q_m is the maximum or peak discharge, in cubic feet per second; K is a constant; t is the time from the beginning of the storm, in minutes, corresponding to any Q_d; and t_l (time lag to peak) is the time corresponding to Q_m. A similar expression was derived by Mr. Richmond T. Zoch.[8] The writer found:

$$C_1 - t = C_2 \log Q \quad\ldots\ldots(3)$$

in which C_1 and C_2 are constants applicable to a particular drainage area; t is the time from the beginning of the storm, in days; and Q is discharge, in cubic feet per second. It may be seen that these are different forms of the same function.

An investigation of the characteristics of this storage run-off function shows that the discharge at any time subsequent to the peak is a direct function of the peak discharge, and that the percentage of the storage yet to run off at any time, and the ratio of the discharge at that time to the peak discharge, is constant irrespective of the magnitude of the storm. As the hydrographs of greater storms yet lie entirely above those of lesser storms this should reconcile the two schools of thought, one of which claims that the length of base of the hydrograph is independent of the magnitude of the storm, and the other of which states, apparently conversely, that the larger the storm, the longer the hydrograph will be. The length of base is actually infinite, but the time until the ordinates have receded to some particular infinitesimal proportion of the peak ordinate is approximately constant for any drainage area.

Another corollary to the logarithmic expression for storage run-off is that discharge is directly proportional to the volume remaining in storage. For example, Equation (3) may also be expressed as,

$$Q = e^{\alpha} \quad\ldots\ldots(4)$$

in which e is the base of Naperian logarithms, and

$$\alpha = \frac{C_1 - t}{C_2} \quad\ldots\ldots(5)$$

The volume, V_s, remaining in valley storage at any time is equal to the area under the curve and to the right of the point corresponding to the time, or,

$$V_s = \int_t^{\infty} e^{\alpha}\, dt = -\, C_2 \left[e^{\alpha}\right]_t^{\infty} = C_2\, e^{\alpha} = C_2\, Q \quad\ldots\ldots(6)$$

A convenient name for the part of the hydrograph occurring up to the time of concentration is "concentration curve." Although it governs the shape of the "storage curve" (or descending limb of the hydrograph), the shape of the concentration curve is itself dependent on several factors. One of these is the "histogram" of the drainage area, a function of the shape of the drainage area, and the arrangement of tributaries. A histogram is a graph on which stream-bed, or "thalweg," distances from a point down stream are expressed as abscissas; and drainage basin widths, or, rather, areas per unit of distance, are expressed as ordinates. If the histogram is altered by substituting for distances the time required for water to travel to the point of measurement, its integral will then become the concentration curve. In other words, the increase in ordinate of the hydrograph per unit of time is proportional to the additional area, water from which first reached the point of measurement during that unit of time. The time of travel itself is dependent on stream velocity. The velocity varies widely, from place to place on the same stream, but was found for the Mississippi River to fluctuate about a value approximately proportional to the one-fourth power of the distance from the head-waters. The time of travel from the head-waters to any point will then be proportional to the three-fourths power of the distance because, if $\frac{dx}{dt} = x^{0.25}$; $dt = x^{-0.25}\,dx$; and, $t = \frac{4}{3}\,x^{0.75}$. It follows, then, that for similar drainage areas the time of concentration may be expected to be proportional to the three-fourths power of the length, or approximately proportional to the three-eights power of the area.

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[8] *Monthly Weather Review*, September, 1934.

It is now time to define a new concept, which will be referred to as the "basic hydrograph." This is the hydrograph resulting from a uniform rain with a duration equal to the time of concentration. This hydrograph has for its rising limb the concentration curve and for its falling limb the storage curve. There is a sharp break at the top. This break has actually been found on continuous gage records (see Fig. 1), although storms of sufficient duration to produce actual basic hydrographs on large drainage areas are quite infrequent, and the supply of data available from water-stage recorders at any place is small.

When a storm with a duration less than the time of concentration occurs, the hydrograph has a somewhat different shape, as shown by Figs. 1 and 2. A method of deriving the hydrographs of storms of shorter duration has been developed. It may also be used to synthesize the basic hydrograph from one of a known shorter duration. The storage curve has been described as conforming to Equation (3). This is only approximately true. Actually, the storage curve is the sum of a theoretically infinite number of curves, each of which conforms closely to the equation, and each of which expresses the run-off from an element of area at a certain time-distance from the point of measurement. A convenient, fairly close approximation is to group the run-off from the area, water from which first reaches the point of measurement within a certain unit of time, and determine the constants of the equation for that group. The unit should be about one-tenth the time of concentration, or less. The smaller the unit the better will be the definition of the curves. In practice, the equation itself is not determined, but only the ratio, λ, of two ordinates one unit of time apart; this is a constant for the particular equation. This ratio (see Appendix I) is found to be:

$$\log_e \lambda = -\frac{h}{V_s} \qquad (7)$$

As V_s is approximately equal to h times the mean time, t_m, it follows:

$$\log_e \lambda = \frac{-1}{t_m} \qquad (8)$$

in which h is the difference between the ordinates on the concentration curve at the beginning and end of the unit of time; and V_s is the area between the corresponding abscissas and to the left of the concentration curve, or the volume of run-off from the corresponding area remaining in valley storage, as shown by Fig. 3. As this is the volume of run-off that has not appeared at the gage by the time of the cessation of the storm, it must be the volume under the storage curve also. Successive ordinates on the storage curve are found by multiplying each ordinate in turn by the ratio, λ. This ratio is determined for each unit of time until the time of concentration.

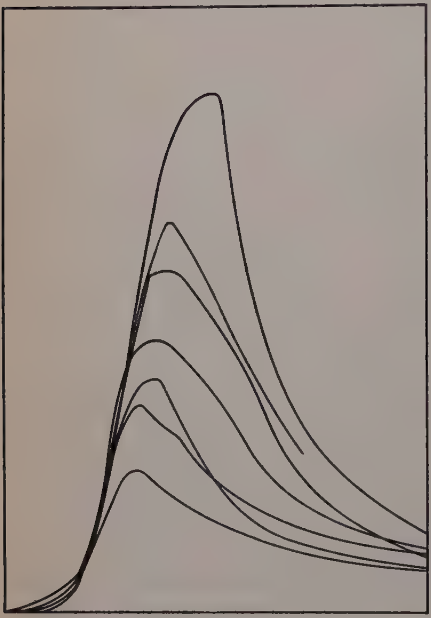

FIG. 1.—GROUP OF HYDROGRAPHS OF PEMIGEWASSET RIVER, AT PLYMOUTH, N. H., REDUCED TO COMPARATIVE SCALE

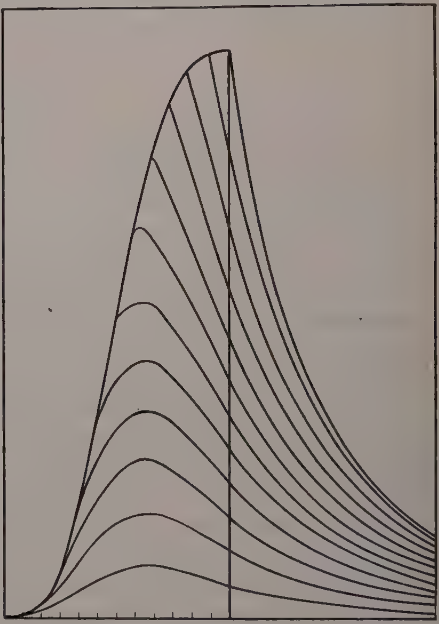

FIG. 2.—HYDROGRAPHS FROM STORMS WITH DURATIONS OF SUCCESSIVE TWELFTHS OF TIME OF CONCENTRATION COMPUTED FROM ASSUMED BASIN HYDROGRAPH

The hydrograph for a storm of any duration will follow the basic hydrograph until the cessation of the storm. At the next unit of time the ordinate will be the sum of the quantities, h, for the preceding units of time, each multiplied by its respective value of the ratio, λ, plus the h-value for the last unit of time multiplied by the ratio of actual run-off volume from its corresponding area to its V_s (which is the ratio of the duration of the storm to the mean abscissa for the unit of time). This last-mentioned multiplication is necessary so that the volume under the corresponding sub-storage curve will be equal to the actual run-off volume available. The following ordinates are the sums of all preceding subordinates multiplied by their respective ratios, plus the *h*-value for the last unit of time treated as in the foregoing text. After the time of concentration is reached this becomes the sum of all the preceding subordinates multiplied by their respective ratios. A sample computation is shown by Appendix II.

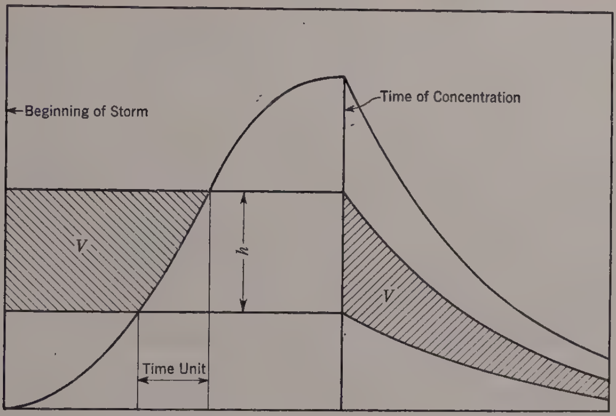

FIG. 3.—VALLEY STORAGE AND SUB-STORAGE CURVE FOR A TIME UNIT

The basic hydrograph may be synthesized from one resulting from a storm of known duration by a reversal of the foregoing process. The height of the peak will be the area under the known hydrograph divided by the duration of the storm, as the peak run-off rate is equal to the rainfall rate multiplied by the percentage run-off. The basic hydrograph will follow the known hydrograph until the cessation of the storm. The aforementioned ratios may be determined in advance, since they can be reduced (as shown) to functions of time. The subordinates for time units previous to the cessation of the storm are treated as described in the foregoing text, and their sum is subtracted from the corresponding ordinate of the hydrograph. The difference is divided by the ratio of the duration of the storm to the mean abscissa of the unit of time. This quotient is added to the last ordinate of the basic hydrograph to give the following one. This procedure is followed until the subtraction gives a negative result. At this point, the ordinate should have reached the maximum previously computed. The storage curve, taken as the sum of all sub-storage curves, then follows.

TRANSPOSITION OF HYDROGRAPHS

As many engineering problems require hydrographs at points where no stream-flow measurements have been taken, some method of transposing hydrographs is necessary. The method proposed by Mr. Sherman, according to which the ordinates and abscissas of the unit hydrograph vary with the square root of the area, has been discussed. Unit hydrographs derived by this method will be correct for total volume of run-off, of course, and will lie within reasonable limits of variation in peak run-off rate; however, they will be subject to considerable error in time of concentration and shape. For several positions on the same stream these hydrographs cannot be used comparatively.

In the course of flood-control studies of the rivers of Maine and New Hampshire, under the United States Engineer Department, it was found necessary to devise some more consistent method of transposing hydrographs. Under the pressure of time, a detailed study being impossible, Mr. Ralph Reinhart, of the Engineer Department, constructed a set of curves based on a large number of distribution graphs which had been prepared by the writer. These curves (shown in Fig. 4) each represent the percentage of the total run-off from a storm of one day duration for one of the first six days of run-off, plotted against drainage area. Each sheet, representing some particular day, contained four curves, representing four groups of drainage area types. It became possible, with experience, to pick the applicable curve after examining topographic maps of the drainage area in question. Each day's proportion of the run-off was then taken from the proper sheet. Only seven sheets were prepared, as the remainder of the hydrograph could be computed quickly by means of the storage curve formula. (The percentage for the seventh day is not shown in Fig. 4.) No attempt has been made to define, exactly, the topography falling in each group, as such an attempt would only be misleading; however, topography producing rapid run-off will fall in a group having a high rate the first few days, and *vice versa*. To determine the run-off distribution for any type of drainage area it will only be necessary to derive the distribution graph for a similar area having run-off records, plot the daily distribution percentages, and find which set of curves fits best. A set of curves of this type is very useful where a large number of hydrographs of a large number of drainage areas is required, and time or the budget does not allow a more refined method.

Certain situations arise where a high degree of refinement in hydrograph computation is required. An example of this would be the investigation preceding the construction of a large flood-control project. In such a case, particularly if hydrographs at various points on a stream are required, Mr. Sherman's transposition does not have a sufficient degree of precision. The aforementioned set of curves (Fig. 4) gives comparable hydrographs for successive points on a stream, but cannot be relied upon for peak run-off rates. A transposition of the basic hydrograph is then the best solution. The basic hydrograph is used in preference to the unit hydrograph because the relation between two basic hydrographs is a function of type and size of drainage area only, whereas the relation between two unit hydrographs involves the ratio of the time unit to the time of concentration, which is not the same for different sized drainage areas. The two unit hydrographs may vary greatly in shape, therefore, whereas the two basic hydrographs would be very similar in shape.

The transposition of the basic hydrograph of one drainage area to another similar area is very simple. The histograms of similar areas are similar, and their concentration curves are very nearly similar. The time of concentration, as shown previously, is approximately proportional to the three-fourths power of the length of the stream, or the three-eighths power of the area. It is pref-

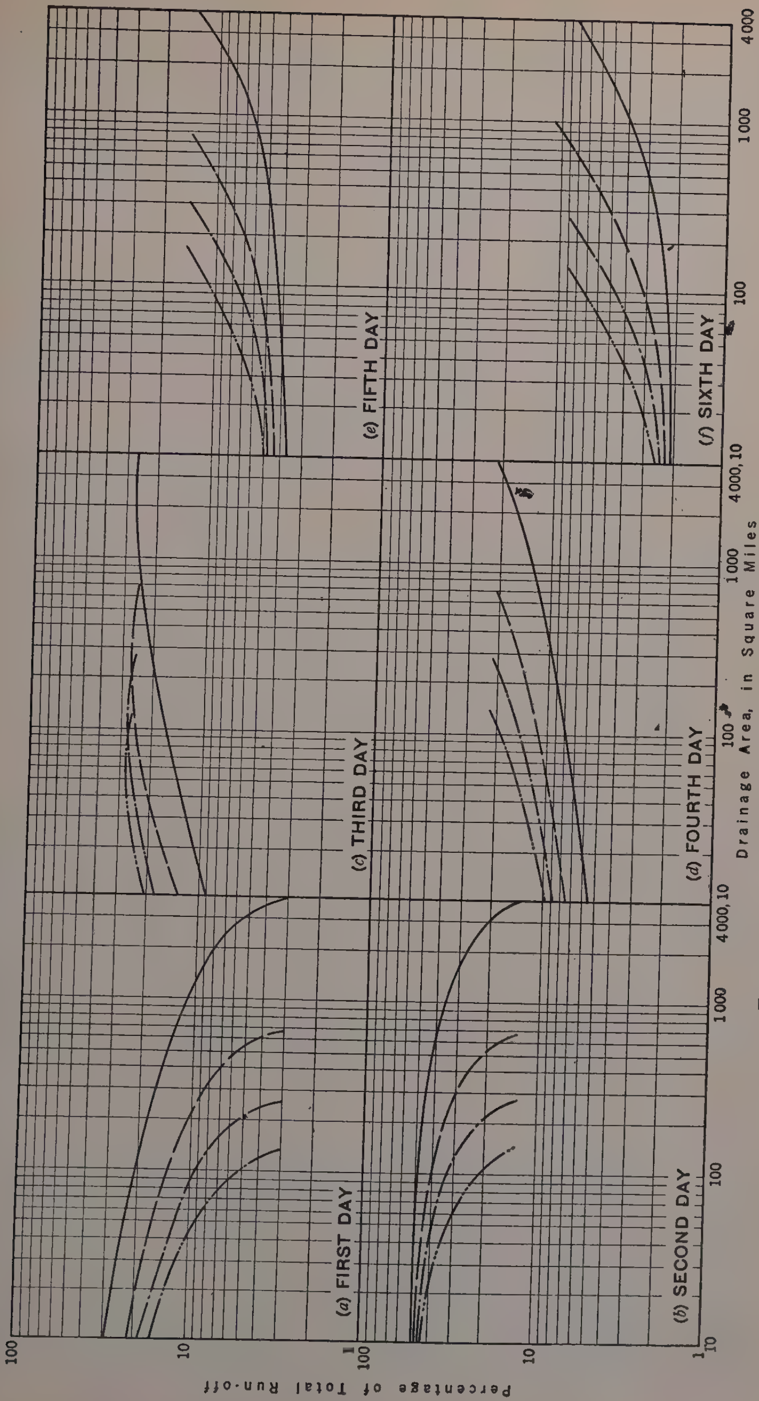

Fig. 4.—Distribution Percentages on Successive Days of Run-Off

erable to use the length if it is available; otherwise the area will suffice. All abscissas will then vary as the time of concentration. As a check, the area under the curve must be the product of the time of concentration and the maximum ordinate. As the height of the basic hydrograph is proportional to the intensity of the storm producing it, the vertical scale must always be such that the total area under the curve is equal to the total volume of run-off; it needs no special treatment in transposition.

If a unit hydrograph is desired it can be computed from the transposed basic hydrograph, as previously described. Distribution graphs for storms of any durations may also be computed by the same method.

Composition or Reconstitution of Hydrographs

The purpose of all the procedures mentioned herein is to make possible the construction of hydrographs from rainfall data, not only for locations where gaging stations exist, but for others as well. For this purpose there must be available methods of constructing the hydrograph resulting from a succession of storms of various magnitudes and durations, separated by various intervals, these methods making use of such tools as unit hydrographs or the proposed basic hydrographs.

One element of flow must be determined independently of rainfall; that is, the "base" or ground-water flow, which may be defined as that part of the run-off which is absorbed by the ground during precipitation or thawing, and which, after traveling varying distances under ground, reaches the beds of streams at some later time. The ground acts as an equalizing reservoir, and the ground-water flow has no characteristics to identify any part of it with any particular period of precipitation. The ground-water curve is that curve which is approached asymptotically by the hydrograph during intervals between storms. It follows an annual cycle, remarkably uniform from year to year. It is generally high during the spring thaws and rains, low in the summer, a little higher in the fall, and low again in the winter. An example is offered by the ground-water curves for five successive years at Mercer, Me., on the Sandy River, shown in Fig. 5(*a*). This ground-water curve may be determined as a mean over several years, and plotted as cubic feet per second per square mile. It may then be used very satisfactorily, for any drainage area in the general region, as a base to which storm hydrographs may be added. If the hydrograph of a particular storm is to be reconstituted, it is obviously preferable to use the actual ground-water curve for that period, if available.

Another element that must be pre-determined in constructing hydrographs is the percentage of run-off. This is the percentage of the precipitation that appears in the streams as run-off within a sufficiently short time to be identified with a particular storm. It may be determined for any storm by dividing the total volume of surface run-off from an area by the total volume of rainfall on that area. The percentage of run-off goes through an annual cycle remarkably similar to that of ground-water flow. When the ground-water level is high the absorptive power of the ground is low, and the percentage of run-off is high. It is, in fact, almost proportional to the ground-water flow. An ex ample of this cycle is given by Fig. 5(*b*), which is the percentage of run-off for the same location and period covered for ground-water by Fig. 5(*a*). Percentages greater than 100 are caused by melting snow.

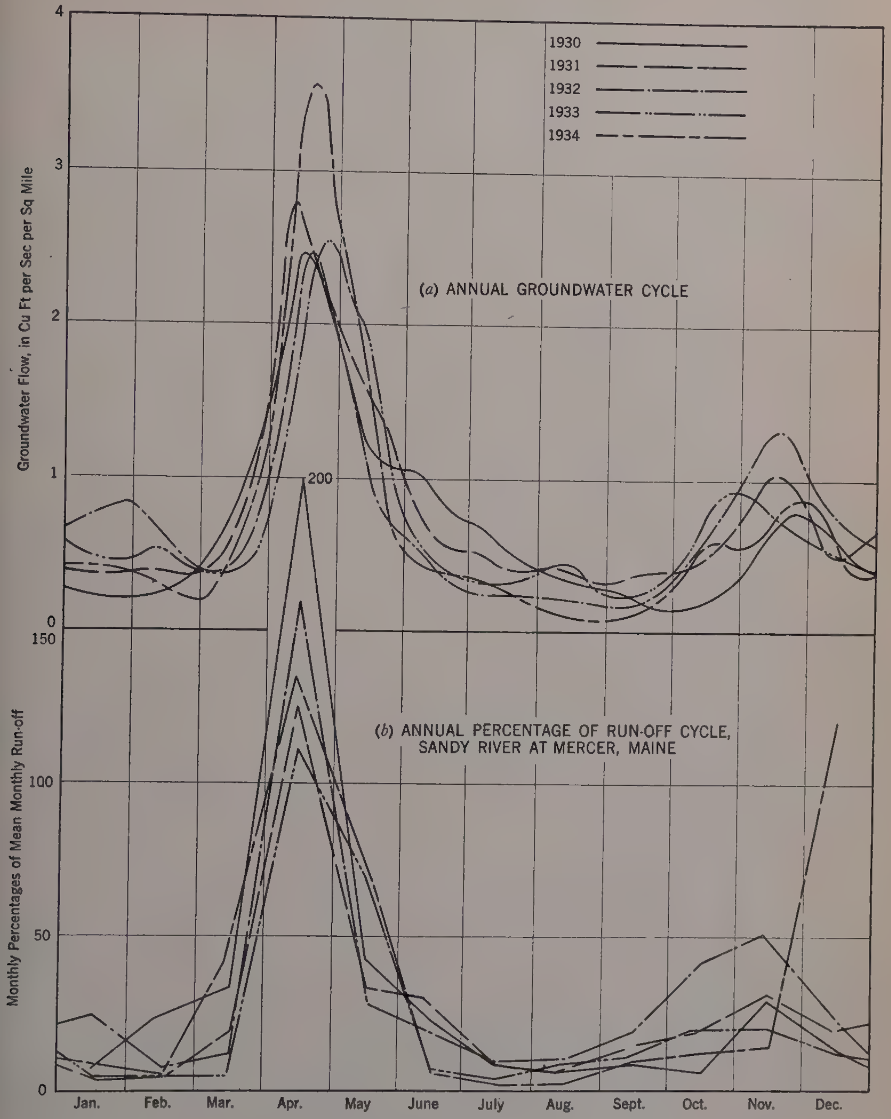

FIG. 5.—CURVES OF SANDY RIVER, AT MERCER, ME.

It is clear that in reconstituting a hydrograph from rainfall the time of year must be known in order to have the correct base flow and the correct run-off percentage for each storm. In composing the hydrograph resulting from an assumed storm, the time of year must be assumed also. Each day's rain must be multiplied by the percentage of run-off for that date, and the resulting hydrograph must be added to the ground-water hydrograph for the period covered.

The generally accepted method of reconstituting the hydrograph resulting from a group of storms is, as mentioned previously, to add, directly, the hydrographs resulting from the rainfall occurring in each unit of time, each one computed as if it were the run-off of an isolated, unit storm, using the distribution graph. The simplicity of this method is its strength. The hydrograph so developed, however, will follow an actual hydrograph only very imperfectly. One particular discrepancy that arises is that for a succession of storms of equal intensities, or a continuous rain, the method will frequently give a run-off rate exceeding the product of the rainfall rate and percentage of run-off. This is particularly true when the time to the peak of the unit hydrograph is very nearly the time of concentration. In such a case continuation of rainfall should not increase the discharge; but the method described adds to the peak the discharge of one day later on the falling limb, plus that of two days later, etc. Mr. Bernard recognized this effect, but attributed it to a flood wave. It is clear from the foregoing analysis that it is only the result of a flaw in the method.

The analysis of the characteristics of the basic hydrograph leads logically to a method of reconstituting hydrographs from rainfall. Underlying this method are two fundamental principles. The first is that the shape of the hydrograph resulting from a single storm is a function of the duration of the storm. The second is that the hydrograph resulting from a steady, continuous rain having superimposed on it a brief, heavy shower will be the sum of the hydrographs that would be produced by the two separately. This will be exactly true only on streams having a vertical stage-velocity relation; however, the similarity between hydrographs of storms of different magnitudes but of equal duration shows that little error is produced by accepting this assumption.

In records of the U. S Weather Bureau precipitation is recorded by days, and where unit hydrographs are the means of computing run-off, the records are used by days. In the proposed method, however, precipitation must be

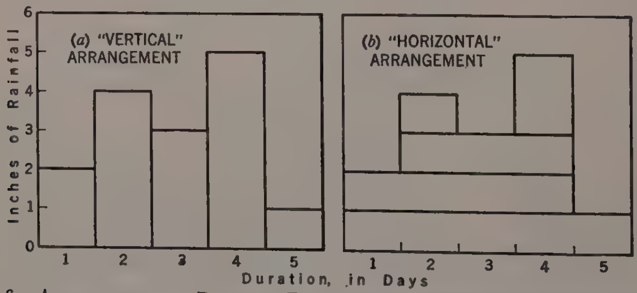

FIG. 6.—ARRANGEMENT OF RAINFALL DATA FOR RECONSTITUTION OF HYDROGRAPHS

arranged "horizontally"; that is, where rainfall occurred on several successive days, it is divided into elements, each of which consists of a certain quantity of rain per day for the maximum number of days. This is more clearly illustrated in Fig. 6. Each of these elements, or "blocks," will produce a hydrograph of different shape. The vertical scale of each hydrograph is proportional to the height of the corresponding block; analysis of the basic hydrograph provides the shape for each duration. These elements of the hydrograph may be added to each other directly until a time is reached when the rainfall rate, after having decreased, again increases, as on Day 4, Fig. 6. When this occurs the ordinate of the sum of the hydrographs resulting from preceding or lower blocks is carried as a constant through the duration of the additional block, and a rate of rainfall sufficient to maintain this flow is subtracted from the total rate during the period of the additional block. The hydrograph produced by the difference in rainfall is then computed and added to the other. At the end of this additional block of precipitation the underlying part of the hydrograph (all of the hydrograph other than that computed for the additional block of rain) reverts to the descending curve, which is the storage curve for the part of the rain that has ceased, plus the uniform flow from the part of the rain that continues. A sample computation is given in Appendix III.

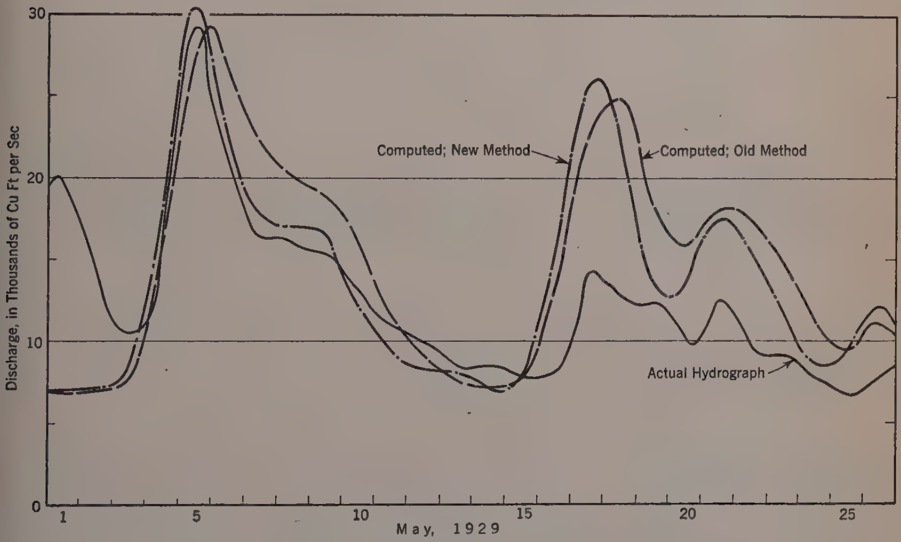

Fig. 7.—Hydrographs of Merrimac River, at Manchester, N. H.

The method described herein will produce a hydrograph which, in shape, will parallel an actual hydrograph, even reproducing minor fluctuations, whereas the hydrograph produced by the older method reproduces only the major undulations, and with a lesser degree of precision. Both methods rely very heavily on correct determination of percentage of run-off and ground-water flow. An example is offered by Fig. 7, which shows an actual hydrograph compared with hydrographs computed by the two methods described.

Conclusions

For studies in which time is short and great accuracy is not essential the use of unit hydrographs or distribution graphs, with their corresponding method of reconstituting a hydrograph, will be the most satisfactory method. In such case distribution curves similar to those shown in Fig. 5 will be found superior in accuracy to the unit hydrographs derived by the transposition method proposed by Mr. Sherman, and also much quicker, if hydrographs at a sufficient number of different locations are required to justify the labor of constructing the curves. Even for rather hasty studies the curves of annual percentage of run-off and ground-water flow will probably be found desirable. Where a high degree of accuracy is required, or for long-continued studies, the proposed methods will be found to be superior. Particularly in forecasting floods the greater degree of refinement should be of value. In such cases, the required curves and tables of daily run-off for various durations of precipitation, may be prepared in considerable detail. At any time, it will then be possible, very quickly, to compute the hydrograph that may be predicted to result from the rain to date. If future rain can be estimated for a day or two a more complete prediction can be made. The coefficients can be corrected from time to time as actual discharges become available.

APPENDIX I

DERIVATION OF THE RATIO OF SUCCESSIVE ORDINATES ON A STORAGE CURVE

As shown in the paper, the storage curve conforms to the general formula expressed by Equation (4); and the volume remaining in valley storage at any time is given by Equation (6). The volume flowing out during any time, t, is:

$$V_t = \int_0^t Q\,dt = \int_0^t e\,\alpha\,dt = -\,C_2\,[e\,\alpha]_0^t \ldots\ldots\ldots\ldots (9)$$

When $t = 0$, the exponent, α (see Equation (5)), becomes,

$$\frac{C_1 - t}{C_2} = \frac{C_1}{C_2} = \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and, from Equation (10),

$$V_0 = C_2\,Q = C_2\,e^\beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

When one-half the volume in storage has run-off, V_t is equal to V_s and equal to one-half the original volume; that is:

$$C_2\,e^\beta - C_2\,e^\alpha = 0.5\,C_2\,e^\beta \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

or,

$$0.5\,C_2\,e^\beta = C_2\,e^\alpha = \frac{C_2\,e^\beta}{e^\epsilon} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which $\epsilon = \frac{t}{C_2}$. By elimination and transposition:

$$e^\epsilon = 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

and

$$\epsilon = \frac{t}{C_2} = \log_e 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

If the time interval required for one-half the volume to run off is designated as t_i, Equation (14*b*) becomes:

$$\frac{t_i}{C_2} = \log_e 2 \quad \text{(15a)}$$

and,

$$t_i = C_2 \log_e 2 \quad \text{(15b)}$$

As the ordinate, h, is equal to Q:

$$V_s = C_2 \, Q = C_2 \, h \quad \text{(16a)}$$

$$C_2 = \frac{V_s}{h} \quad \text{(16b)}$$

and,

$$t_i = \frac{V_s \log_e 2}{h} \quad \text{(16c)}$$

Let $\frac{1}{t_i} = \lambda = \frac{h}{V_s \log_2 2}$; then, the ratio of the two ordinates, one unit of time apart, is:

$$\lambda = \frac{1}{0.5^{\lambda}} \quad \text{(17)}$$

that is,

$$\log_e \lambda = \frac{h \log_e 0.5}{V_s \log_e 2} = - \frac{h}{V_s} \quad \text{(18)}$$

APPENDIX II

SAMPLE COMPUTATION, ANALYSIS OF THE CONCENTRATION CURVE

Table 1 shows a sample computation form, in this case for the analysis of the concentration curve shown in Fig. 2. The time, y, and the dy-columns are self-explanatory. The values in Column (3) were computed from Equation (17). In the sub-storage curve for each time unit the first ordinate is dy; each ordinate is multiplied by the ratio shown to obtain the succeeding ordinate. In computing hydrographs for storms of short duration, the ordinates of each sub-curve are multiplied by the ratio of the duration of the storm to the mean time for the respective time unit (for Unit 3, Table 1, the mean time is 2.5; for Unit 7, it is 6.5, etc.), except where the time unit is prior to the cessation of the storm, in which case the full value is used. Thus, in a storm lasting four units, Curves (Units) 1 to 4 are used full value, Curve 5 is multiplied by $\frac{4}{4.5}$, Curve 6 by $\frac{4}{5.5}$, Curve 7 by $\frac{4}{6.5}$, etc. Run-off from each unit first appears at the time expressed by its number, and is at the maximum rate as long as the storm lasts, following the descending curve thereafter; or, if the storm has already ceased before its appearance, it follows immediately the curve, reduced proportionately as previously explained.

TABLE 1.—ANALYSIS OF A CONCENTRATION CURVE

Time unit	y (1)	dy (2)	Ratio, λ (3)	SUCCESSIVE ORDINATES (4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	1.1	1.1	0.195	1.1	0.2	....	....	....	....	....	....
2	4.2	3.1	0.520	....	3.1	1.6	0.8	0.4	0.2	0.1	0.1
3	11.4	7.2	0.676	....	....	7.2	4.9	3.3	2.2	1.5	1.0
4	29.5	18.1	0.756	....	....	....	18.1	13.7	10.3	7.8	5.9
5	53.3	23.8	0.803	....	....	....	....	23.8	19.1	15.4	12.3
6	80.0	26.7	0.837	....	....	....	....	....	26.7	22.3	18.7
7	103.8	23.8	0.859	....	....	....	....	....	....	23.8	20.4
8	123.7	19.9	0.878	....	....	....	....	....	....	....	19.9
9	137.4	13.7	0.892	....	....	....	....	....	....	....	....
10	145.7	8.3	0.911	....	....	....	....	....	....	....	....
11	150.3	4.6	0.9245	....	....	....	....	....	....	....	....
12	151.6	1.3	0.931	....	....	....	....	....	....	....	....

TABLE 1.—(*Continued*)

Time unit	SUCCESSIVE ORDINATES (12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
1	....	....	....	....	....	....	....	....	....	....	....	....
2	....	....	....	....	....	....	....	....	....	....	....	....
3	0.7	0.5	0.3	0.2	0.1	0.1	0.1	....	....	....	....	....
4	4.5	3.4	2.5	1.9	1.5	1.1	0.8	0.6	0.5	0.4	0.3	0.2
5	9.9	8.0	6.4	5.1	4.1	3.3	2.7	2.1	1.7	1.4	1.1	0.9
6	15.6	13.0	10.9	9.1	7.6	6.4	5.3	4.5	3.7	3.1	2.6	2.2
7	17.5	15.0	12.9	11.1	9.5	8.2	7.0	6.0	5.2	4.4	3.8	3.3
8	17.5	15.3	13.5	11.8	10.3	9.1	8.0	7.0	6.1	5.4	4.7	4.2
9	13.7	12.2	10.9	9.7	8.7	7.7	6.9	6.1	5.5	4.9	4.3	3.9
10	....	8.3	7.6	6.9	6.3	5.7	5.2	4.7	4.3	3.9	3.6	3.3
11	....	....	4.6	4.3	3.9	3.6	3.4	3.1	2.9	2.6	2.4	2.3
12	....	....	....	1.3	1.2	1.1	1.0	1.0	0.9	0.8	0.8	0.7

The same form shown in Table 1 is used in the synthesis of the concentration curve from hydrographs of storms of known duration, by the procedure described in the paper.

APPENDIX III

SAMPLE COMPUTATION OF RECONSTITUTED HYDROGRAPH

Appendix III offers an example of the computation of a hydrograph from rainfall data by means of the basic hydrograph. In this example, the rainfall shown in Fig. 6 will be used. It will be assumed that the drainage area for which the hydrograph is to be constructed has a space of 100 sq miles; a groundwater flow at this season of 3 cu ft per sec per sq mile; and a run-off factor at this season of 50 per cent. It will be further assumed that its basic hydrograph is known and has a time of concentration of three days, and that hydrographs for storms of one day and three days duration have been reduced to the distributions shown in Table 2.

TABLE 2.—DISTRIBUTION, IN PERCENTAGE OF DAILY RUN-OFF RATE

Description	(a) SUCCESSIVE DAYS; ONE-DAY STORM						(b) SUCCESSIVE DAYS; THREE-DAY STORM					
	1	2	3	4	5	6	1	2	3	4	5	6
Distribution	10	34	28	14	7	3.5	10	45	85	80	40	20

Stream flow on any later day will be one-half that on the preceding day. Flow on the fourth day of a longer storm will become 100% of the daily run-off rate, remaining at 100% for the duration of the storm, then going to 80%, 40%, etc., as in the case of the three-day storm (Table 2(*b*)).

A flow of 26.89 cu ft per sec for one day will be produced by 1 in. of run-off from 1 sq mile. The run-off, at 50% run-off factor, from 1 in. of rain per day on 100 sq miles is, therefore, 1 345 cu ft per sec. The rainfall shown on Fig. 6, being, on successive days, 2, 4, 3, 5 in., and 1 in., respectively, is the equivalent of: A five-day storm at 1 in. per day beginning on Day 1; a four-day storm at 1 in. per day beginning on Day 1; a three-day storm at 1 in. per day beginning on Day 2; a one-day storm of 1 in. beginning on Day 2; and a one-day storm of 2 in. beginning on Day 4.

TABLE 3.—COMPUTATION OF RECONSTRUCTED HYDROGRAPH: STANDARD PROCEDURE (UNITS ARE CUBIC FEET PER SECOND)

Day	DAILY RUN-OFF; STORMS AT 1 INCH PER DAY*					Total surface run-off	Ground-water flow	Total run-off
	Five-day storm (1)	Four-day storm (2)	Three-day storm (3)	One-day storm (4)	One-day storm* (5)	(6)	(7)	(8)
1	135	135	....	....	....	270	300	570
2	607	607	135	135	....	1 484	300	1 784
3	1 140	1 140	607	458	....	3 345	300	3 645
4	1 345	1 345	1 140	376	269	4 475	300	4 775
5	1 345	1 076	1 076	188	916	4 601	300	4 901
6	1 076	538	538	94	752	2 998	300	3 298
7	538	269	269	47	376	1 499	300	1 799
8	269	134	134	24	188	749	300	1 049
9	134	67	67	12	94	374	300	674
10	67	34	34	6	47	188	300	488

* Column (5), Table 3, is the equivalent rainfall in a one-day storm of 2 in. on Day 4.

TABLE 4.—COMPUTATION OF RECONSTRUCTED HYDROGRAPH: SPECIAL PROCEDURE (UNITS ARE CUBIC FEET PER SECOND)

Day	HYDROGRAPHS NOT MODIFIED		MODIFIED HYDROGRAPHS		Total run-off
	Preceding storm (1)	Succeeding storm (2)	Preceding storm (3)	Succeeding storm (4)	(5)
1	1 000	...	1 000	...	1 000
2	600	...	600	...	600
3	360	100	463	23	486
4	216	350	463	80	543
5	130	600	463	137	600
6	78	600	463	137	600
7	47	466	360	106	466
8	28	280	216	64	280
9	17	168	130	38	168
10	....	...	....	...	....

The computation is set up in Table 3. The run-off from the foregoing equivalent storms is shown in the columns headed "Daily Run-Off" (Columns (1) to (5)), each equivalent storm to one column, in the order given.

If an increase in rainfall should come when the hydrograph is falling, a special procedure is used. For example, assume a hydrograph to be falling after the cessation of a storm, with a ratio between successive ordinates equal to 0.60, as shown in Table 4 (Column (1)). A following storm, of intensity sufficient to produce a maximum run-off of 600 cu ft per sec, begins on Day 3, its hydrograph, if it had occurred alone, being as shown by Column (2). To combine the two, the ordinate on Day 2 is multiplied not by the ratio, 0.60, but by its square root, 0.77 (giving the ordinate at the beginning of Day 3 instead of at the middle of Day 3). This ordinate continues through the following storm, the ordinate that would have occurred on Day 3 now occurring on Day 7, the day following the end of the following storm, as shown in Column (3), Table (4). The increase in run-off under this hydrograph comes out of the run-off volume of the following storm, which must be reduced accordingly, so the modified hydrograph (Column (4), Table (4)) is in the ratio to that in Column (2) of $\frac{600 - 463}{600}$. The total run-off is the sum of Column (3) and Column (4), Table 4.

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PAPERS

TESTS ON BUILT-UP COLUMNS OF STRUCTURAL ALUMINUM ALLOYS

By M. Holt,[1] Jun. Am. Soc. C. E.

Synopsis

Results from tests on fifteen built-up column specimens of structural aluminum alloys and on two of structural steel are presented in this paper. The investigation was a study of the effect of the shape of section on the strength of built-up columns. All specimens had maximum slenderness ratios of about 40 and cross-section areas of 7 or 8 sq in. Strains and deflections were measured under various loading conditions. In the range of working loads the shape of the cross-section has no significant effect on the ratio of the measured to computed deflections of the axis of the specimen nor on the magnitude of the transverse shear indicated by the lacing-bar stresses. The shape of the cross-section has a significant effect on the ratio of the measured to computed stresses, the local deflections under higher loads, and on the type of failure, whether by buckling of a component part or by bending as a unit. Short built-up columns of high-strength aluminum alloys are likely to fail by local buckling at average stresses less than the yield strength of the material when the width-to-thickness ratio of outstanding plates and legs of angles exceeds about 12 and when the width-to-thickness ratio of the cover-plates exceeds about 40.

Introduction

Structural designers are always confronted with the problem of determining the strength of compression members. These members may be single units (structural shapes), or they may be built-up of a number of such shapes. Considerable work, both analytical and experimental, has been done in the study of the strength of columns. It has been established that the strength of the member as a column is affected by its stiffness and the compactness of the cross-section, as well as by the strength of the material. Columns acted upon by axially applied loads might fail at stresses considerably less than the

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January 15, 1939.**

[1] Research Engr., Aluminum Research Laboratories, Arnold, Pa.

yield strength of the material by sidewise bending because of lateral instability of the member as a whole, or by twisting or buckling of a component part because the parts themselves are not sufficiently rigid. The solution of the problem then lies in knowing not only the fundamental behavior of columns, but also the effects of the shape and proportions of the cross-section of the column upon its compressive strength.

The results of many tests on steel columns can be found in the technical literature. These test results form the basis for specification requirements. The tests discussed in this paper were made as part of a research program to acquire information for the establishment of a set of specifications for the structural use of the high-strength aluminum alloys.

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

DESCRIPTION OF SPECIMENS

Fifteen of the seventeen specimens used in this investigation were fabricated of high-strength aluminum alloys having mechanical properties comparable with those of mild steel. The other two specimens were of structural steel and duplicated two of the aluminum alloy specimens in size and shape of the cross-section. Each of the specimens had a greatest slenderness ratio of about 40 and an area of 7 or 8 sq in.

Table 1 contains a description of the specimens. Each piece of material was given a number (Column (8), Table 1) by means of which specimens for determining the mechanical properties of the materials were identified. The weight of the details, such as rivets, lacing-bars, and batten-plates, is the difference in the weight of the complete specimen and that of the main members before the rivet holes were punched. The length was determined after the ends had been carefully machined by turning the specimen on centers in a lathe. The gross area was calculated from the weight and length of short pieces and the nominal specific gravity of the material, 2.79 for Items Nos. 17 S and 25 S, and 7.85 for steel. The aluminum alloys were in the heat-treated condition.[2] The radii of gyration were computed on the assumption of unified action of the main members. The slenderness ratios of the specimens as a whole are given for the two principal axes, as is the slenderness ratio of the segments, which is the distance between two adjacent rivets divided by the radius of gyration of the part (see Columns (17), (18), and (19), Table 1).

In order to emphasize the effect of the shape of the cross-section and to obtain a suitable area of cross-section, the component parts used in some of these specimens are thinner than allowed by specifications commonly used in the design of structural steel.

The mechanical properties of the materials in tension are given in Table 2. The tests were made on A. S. T. M. (American Society for Testing Materials) standard tensile specimens having a rectangular section and a width of 0.5 in. The stress-strain relations of the aluminum alloys were determined with a mirror-type extensometer, using a 2-in. gage length, whereas a modified form of the bell-crank type of extensometer was used with the steel.

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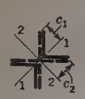
[2] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 7, p. 1279.

TABLE 1.—DESCRIPTION OF TEST SPECIMENS; BUILT-UP COLUMNS



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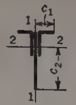

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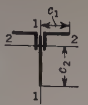

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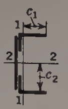
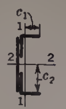


2A1P-3



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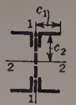

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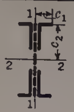

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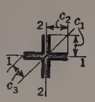

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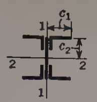

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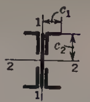

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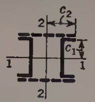
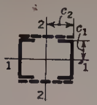


2C-1



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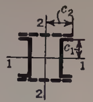

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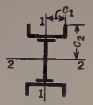

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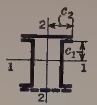

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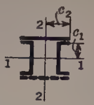

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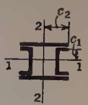


2C2P-1

SPECIMENS		DISTANCE TO EXTREME FIBER, IN INCHES		MAKE-UP			Piece No.	WEIGHT, IN POUNDS			Percentage of weight in details	Length, in inches	Gross area from weight, in square inches	RADII OF GYRATION		SLENDERNESS RATIO			RATIO OF WIDTH TO THICKNESS:		Rivet spacing parallel to stress	Length of lacing-bar
No.*	Type†	c_1	c_2	Piece	No.	Size, in inches		Main member	Complete specimen	Details				k_1	k_2	Column Axis 1	Column Axis 2	Segment	Outstanding legs	Plates		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
2A-1	17S	3.54	3.67	Angles‡	2	$5 \times 5 \times \frac{3}{8}$	156	56.8	63.5	6.7	10.6	79.3	7.00	1.92	2.40	41.3	33.0	28.3	13.3	..	....	..
2A1P-1	25S	3.63	7.32	Angles Plates	2 1	$5 \times 3\frac{1}{2} \times \frac{5}{16}$ $10 \times \frac{1}{4}$	132, 133 144	37.4	38.1	0.7	1.8	49.4	7.67	1.14	2.66	43.3	18.6	..	11.2	25.2	$9.6\, t_a = 12\, t_p$	..
2A1P-2	25S	5.13	7.82	Angles Plates	2 1	$5 \times 3\frac{1}{2} \times \frac{5}{16}$ $10 \times \frac{1}{4}$	132, 133 168	57.1	58.5	1.4	2.4	75.0	7.67	1.89	2.70	39.7	27.8	..	16.0	31.2	$9.6\, t_a = 12\, t_p$	..
2A1P-3	25S	4.00	5.00	Angles Plates	2 1	$5 \times 3\frac{1}{2} \times \frac{5}{16}$ $10 \times \frac{1}{4}$	130, 131 143	46.9	48.5	1.6	3.3	61.4	7.68	1.52	3.91	40.4	15.7	..	16.0	24.0	$9.6\, t_a = 12\, t_p$	..
2A1P-4	25S	3.00	5.00	Angles Plates	2 1	$5 \times 3\frac{1}{2} \times \frac{5}{16}$ $10 \times \frac{1}{4}$	130, 131 143	28.1	29.0	0.9	3.1	37.5	7.68	0.94	3.52	39.9	10.7	..	11.2	12.0	$9.6\, t_a = 12\, t_p$	..
4A-1	25S	4.25	6.00	Angles	4	$4 \times 3 \times \frac{1}{4}$	134, 135 162,‡ 163‡	50.1	68.5	18.4	26.9	72.8	6.82	1.93	5.36	37.7	13.6	16.4	16.0	..	....	40 t
4A-2	25S	3.25	6.00	Angles	4	$4 \times 3 \times \frac{1}{4}$	134, 135 162, 163	36.9	54.0	17.1	31.7	53.6	6.82	1.31	4.95	40.9	10.8	12.5	12.0	..	....	32 t
4A-3	17S	4.00	4.00	Angles	4	$4 \times 4 \times \frac{1}{4}$	160, 161‡	53.8	55.0	1.2	2.2	67.6	7.87	1.63	1.63	41.5	41.5	..	8.0	..	12 t	..
4A1P-1	17S 25S	3.13	4.00	Angles‡ Plates	4 1	$3 \times 2\frac{1}{2} \times \frac{1}{4}$ $8 \times \frac{1}{4}$	157, 167 148	33.1	33.8	0.7	2.1	46.5	7.19	1.17	3.17	39.7	14.7	..	12.0	21.0	12 t	..
4A1P-2	17S 25S	2.63	4.00	Angles‡ Plates	4 1	$3 \times 2\frac{1}{2} \times \frac{1}{4}$ $8 \times \frac{1}{4}$	157, 167 148	26.8	27.4	0.6	2.2	37.5	7.19	0.90	3.02	41.7	12.4	..	10.0	18.0	12 t	..
2C-1	25S	3.50	4.57	Channels	2	7 × 0.314§	136, 137	78.4	98.0	19.6	20.0	108.4	7.17	2.60	2.95	41.7	36.7	15.8	..	..	....	34 t
2C-2	25S	3.50	4.88	Channels	2	7 × 0.314§	138, 139	78.0	98.0	20.0	20.4	108.4	7.17	2.60	4.39	41.7	24.7	15.8	..	..	....	34 t
2C-3	Steel	3.50	4.57	Channels	2	7 × 0.314§	169	22.0	27.4	5.4	19.7	108.3	7.12	2.60	2.95	41.7	36.7	15.8	..	..	....	34 t
2C1I-1	17S	2.50	4.25	Channels‡ I-beam	2 1	5 × 0.190§ 5 × 0.210§	158, 159 155	44.4	45.5	1.1	2.4	61.4	7.18	1.54	2.65	39.9	23.2	..	..	..	15.8 t	..
2C1P-1	25S	4.33	5.00	Channels Plate	2 1	7 × 0.230§ $10 \times \frac{3}{16}$	140, 141 145	87.5	98.5	11.0	11.2	108.7	7.98	2.81	3.36	38.7	32.4	..	..	45.3	16.0 t	40 t
2C1P-2	Steel	4.34	5.00	Channels Plate	2 1	7 × 0.210§ $10 \times \frac{3}{16}$	170 171	23.3	25.8	2.5	10.3	108.7	7.57	2.79	3.35	39.0	32.4	..	..	45.3	16.0 t	40
2C2P-1	17S 25S	2.69	5.00	Channels Plate	2 2	5 × 0.190§ $10 \times \frac{3}{16}$	158, 159 146A, 147	68.3	69.5	1.2	1.7	85.5	7.87	2.29	3.37	37.3	25.4	..	..	46.7	16.0 t	..

* A = angles; P = plates; and C = Channels. † All heat treated. See *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 7, p. 1279. ‡ Extruded; all other sections rolled. § Web.

The mechanical properties of the materials in compression are also given in Table 2. The tests were made on short sections of the pieces having slenderness ratios of 10. The stress-strain relations were determined by measuring the relative movement of the heads of the testing machine with two dial-gages reading directly to 0.001 in.

TABLE 2.—MECHANICAL PROPERTIES OF MATERIALS IN BUILT-UP COLUMNS

Piece No.*	TENSILE PROPERTIES, IN POUNDS PER SQUARE INCH			Percentage elongation in 2 inches	Piece No.*	COMPRESSIVE PROPERTIES, IN POUNDS PER SQUARE INCH		
	Ultimate strength	Yield strength§	Propor-tional limit			Ultimate strength¶	Yield strength¶	Propor-tional limit
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
143†	59 670	43 300	34 000	21.0	130	34 280	34 000	24 000
143‡	57 040	38 600	24 000	19.5	131	34 130	33 800	22 000
144†	55 080	38 200	24 000	17.0	132	40 470	35 000	22 000
144‡	58 000	41 000	24 000	18.5	133	40 800	35 400	22 000
145†	58 750	41 100	36 000	20.0	134	37 310	37 100	28 000
145‡	55 640	37 000	26 000	17.5	135	38 710	38 500	28 000
146A†	57 530	38 900	28 000	19.0	136	40 490	32 000	20 000
146A‡	57 020	38 500	26 000	16.5	137	41 610	33 500	20 000
147†	63 040	46 500	39 000	17.0	138	38 480	32 200	20 000
147‡	58 850	40 900	28 000	18.0	139	40 380	31 950	19 000
148†	59 240	43 700	32 000	18.5	140	39 870	34.000	24 000
148‡	58 680	42 500	32 000	16.5‖	141	40 660	34 700	24 000
161	56 360	33 800	24 000	22.0	155	55 910	42 600	28 000
162	55 900	33 400	22 000	21.0	156	38 450	38 300	26 000
163	54 960	34 000	24 000	20.5	157	36 250	32 000	20 000
167	58 130	41 800	36 000	25.0	158	47 470	37 600	20 000
168†	57 630	39 200	32 000	20.0	159	46 900	38 300	22 000
168‡	57 400	36 300	22 000	19.0	159A	48 040	42 600	26 000
169	62 250	....	31 100	37.0	160	32 450	32 200	24 000
170	60 170	42 000	38 300	33.0	161	32 520	32 200	24 000
171†	62 780	52 500	53 350	25.0	162	33 650	33 500	24 000
....	....	....	....	....	163	34 010	33 500	24 000
....	....	....	....	....	167	39 030	36 900	26 000
....	....	....	....	....	169	52 040	40 800	31 500
....	....	....	....	....	170	40 520	37 200	32 000

* See Table 1. † Specimen taken parallel to direction of rolling. ‡ Specimen taken across direction of rolling. § For aluminum alloys, at 0.2% set; and, for steel, at 0.45% set. ‖ Broke through the gage point. ¶ Slenderness ratio = 10, tested with flat ends.

It should be borne in mind that the aluminum alloys, like other non-ferrous metals and some high-strength steels, do not have a definite point of yielding as does mild steel.

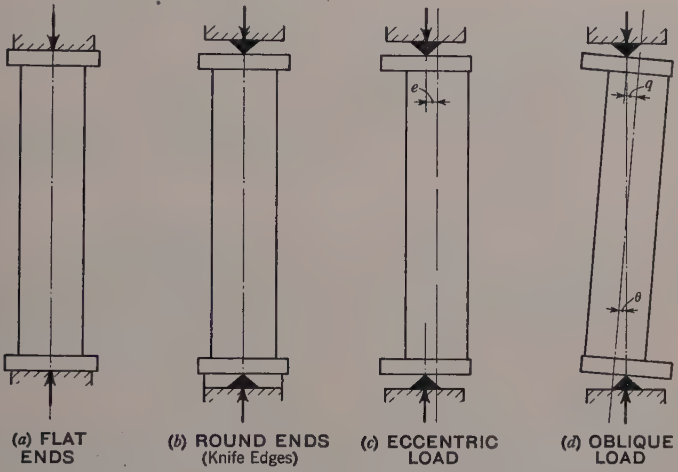
METHODS OF LOADING

The column tests were made in a testing machine of the hydraulic type, having a maximum capacity of 300 000 lb, and intermediate load-ranges. The load was applied to the ends of the specimens using the various loading conditions shown in Fig. 1.

Flat end conditions (Fig. 1(*a*)) were obtained by centering the specimen on the fixed bearing heads of the testing machine.

Round end conditions (Fig. 1(*b*)) were produced with knife-edge bearing blocks consisting of a bearing plate supported by a single knife-edge. The specimens were thus allowed to deflect freely normal to the plane of the knife-edges, but were restrained with the condition of flat ends in the plane of the knife-edges. The specimens were carefully centered in an attempt to make the axis of the specimen coincide with the plane of the knife-edges.

Eccentric loadings (Fig. 1(*c*)) were produced with the knife-edge bearings by placing the specimen off center the same distance at each end and in the same direction. The eccentricity, e, is defined as the distance between the axis of the column and the plane of the knife-edges. The values of the eccentricity ratio, $\frac{e\,c}{k^2}$, were between about 0.4 and 0.6. In computing these values of eccentricity ratio, the value of c, distance from the neutral axis to the point where the stress is desired, was taken as the distance to the extreme fiber and k is the corresponding radius of gyration.



(*a*) FLAT ENDS (*b*) ROUND ENDS (Knife Edges) (*c*) ECCENTRIC LOAD (*d*) OBLIQUE LOAD

FIG. 1.—END CONDITIONS FOR LOADING COLUMNS

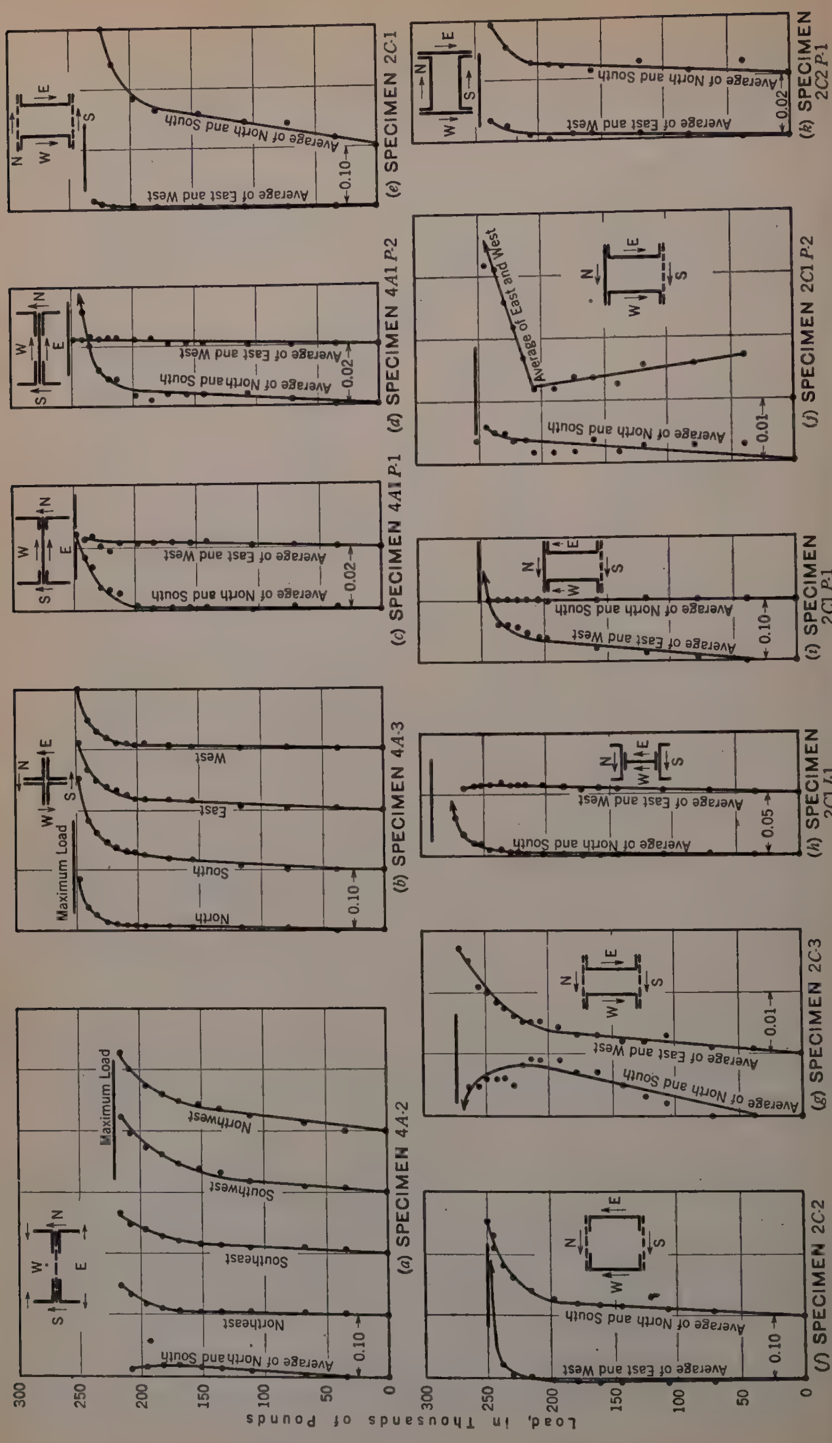
Oblique loadings were produced with the knife-edge bearings by placing the specimen off center the same distance at each end, but in opposite directions at the two ends. The obliquity, q, is defined as the distance between the axis of the column at the ends and the plane of the knife-edges. The values of the obliquity ratio, $\frac{q\,c}{k^2}$, were between about 0.4 and 0.6

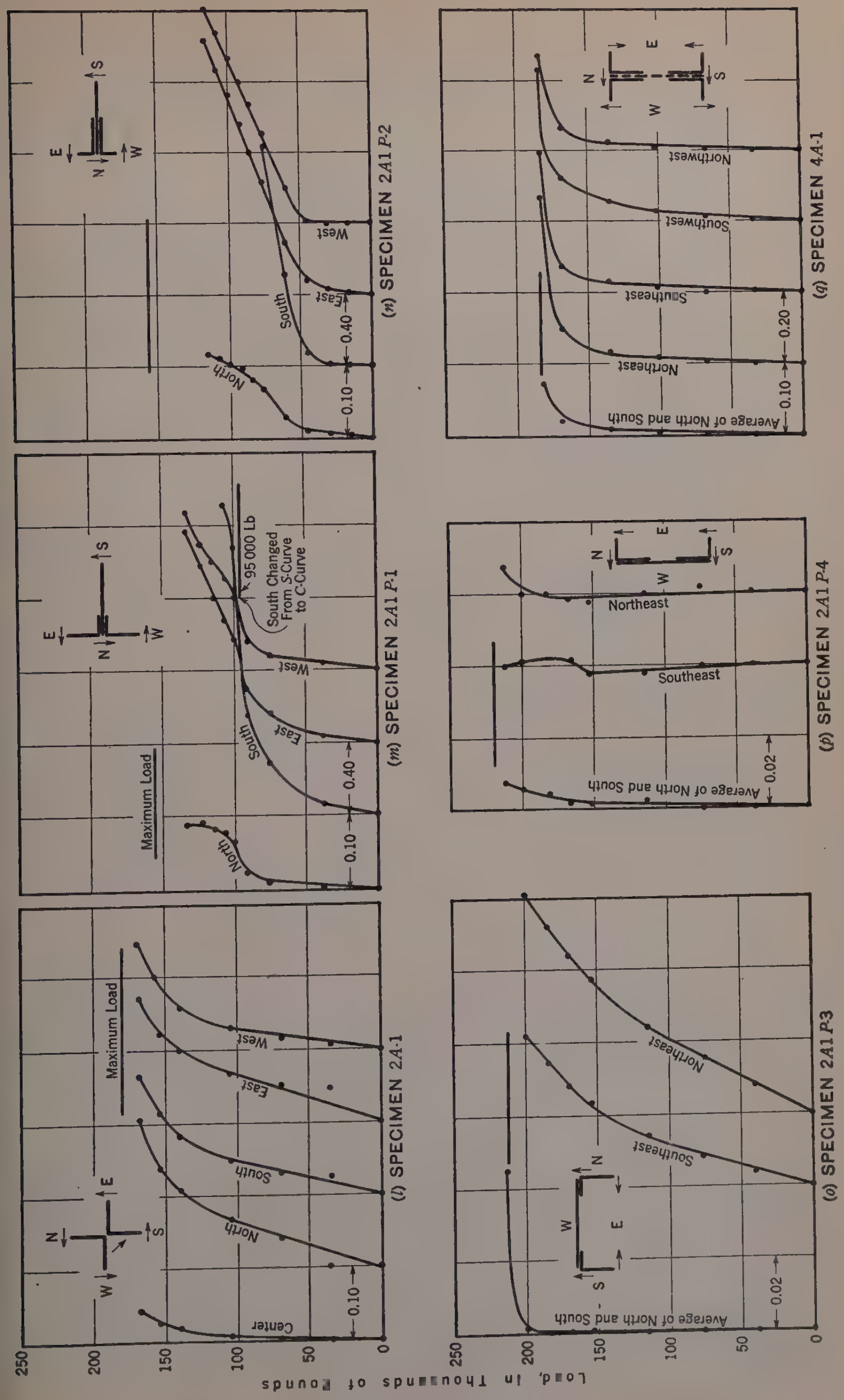
Some tests were made with the specimen off center each way from the plane of the knife-edges. In some cases, the knife-edges were parallel, in turn, to the two principal axes of the cross-section of the specimen. All specimens were loaded to failure with the condition of flat ends, as shown in Fig. 1(*a*).

MEASUREMENTS FOR STRAIN AND DEFLECTION

Strains were measured on several gage lines on the main members using a 2-in. strain-gage. The lines were placed to show the distribution of stress (strain) in the members not only around the section but also along the length of the specimen. Tensometers (1-in. gage length) were used in measuring strains on both sides of the lacing-bars and on rosettes (gage lines intersecting at a point) on batten and cover-plates.

Deflections at the center of the specimens were measured by the use of fine wires, stretched between pins at the ends of the specimens, and scales (graduated


(*a*) SPECIMEN 4A-2
(*b*) SPECIMEN 4A-3
(*c*) SPECIMEN 4A1 P-1
(*d*) SPECIMEN 4A1 P-2
(*e*) SPECIMEN 2C-1
(*f*) SPECIMEN 2C-2
(*g*) SPECIMEN 2C-3
(*h*) SPECIMEN 2C1 I-1
(*i*) SPECIMEN 2C1 P-1
(*j*) SPECIMEN 2C1 P-2
(*k*) SPECIMEN 2C2 P-1



(*l*) SPECIMEN 2A·1

(*m*) SPECIMEN 2A1 P·1

(*n*) SPECIMEN 2A1 P·2

(*o*) SPECIMEN 2A1 P·3

(*p*) SPECIMEN 2A1 P·4

(*q*) SPECIMEN 4A·1

FIG. 2.—RELATION BETWEEN LOAD AND CENTER DEFLECTION (DEFLECTION, IN INCHES)

to $\frac{1}{50}$ in.) mounted on mirrors which, in turn, were mounted on the specimen. In addition to the measurements at the center, the deflections in the direction of bending were measured at the quarter-points in some of the tests using eccentric and oblique loadings.

DISCUSSION OF TEST DATA

The measured strains were interpreted into stresses using values of modulus of elasticity of 10 300 000 lb per sq in. for the aluminum alloys and 29 000 000 lb per sq in. for steel. Only a very small part of all the data accumulated in these tests are presented herein. Fig. 2 gives the load-deflection data from the flat end tests; Fig. 3 gives typical eccentricity-deflection data from the eccentric load tests; and Fig. 4 gives typical transverse shear-obliquity data from the oblique load tests. The curves indicated by the stress in Bars 20 and 32 are shown as broken lines, and the curve indicated by the stress in all bars is shown solid.

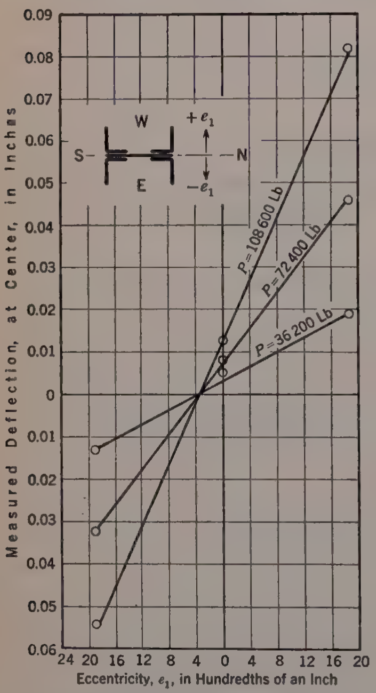


FIG. 3.—RELATION BETWEEN DEFLECTION AND ECCENTRICITY, e_1, SPECIMEN 4A1P-1

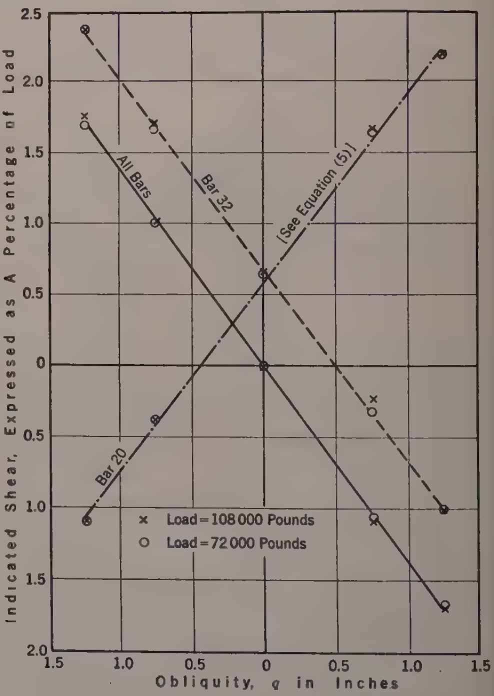


FIG. 4.—TRANSVERSE SHEAR INDICATED BY THE MEASURED STRESSES IN LACING-BAR (25S)

MEASURED STRESSES

In general, the measured stresses are slightly lower than the computed values obtained by the following equations:

For axial loads,

$$s = \frac{P}{A} \quad \text{(1a)}$$

for eccentric loads,

$$s = \frac{P}{A}\left(1 + \frac{e\,c}{k^2} \sec \frac{L}{2\,k}\sqrt{\frac{P}{A\,E}}\right) \quad \text{(1b)}$$

and, for oblique loads,

$$s = \frac{P}{A}\left(1 + \frac{q\,c}{k^2}\right) \quad \text{(1c)}$$

in which s = computed unit stress, in pounds per square inch; P = total load, in pounds; A = cross-section area of main members, in square inches; E = the modulus of elasticity of the material, in pounds per square inch; and L = length, in inches. The agreement was improved when added precautions were taken to avoid the effects of temperature variations on the measured strains.

For the case of low axial loads, there seems to be no significant tendency for any one general type of specimen to have a more uniform distribution of stress than any other type. For higher loads, however, there is a decided tendency for the specimens with thin outstanding plates to have a pronounced variation in the stress distribution. These same remarks can be made for the cases of eccentric and oblique loads when considering the variation of stresses on gage lines the same distance from the neutral axis.

For the case of axial loads, there is no significant variation of stress at the various sections along the length of the member. With eccentric loads the larger stresses are found, of course, on the same side of the specimen at all sections and the variation along the length of the specimen is not great. With oblique loads there is a systematic variation of stress along the length of the member. At one end the greater stresses are on one side of the section and at the other end the greater stresses are on the other side. The stresses at the center section were fairly uniformly distributed.

DEFLECTIONS WITH FLAT-END CONDITIONS

The relations between the load and the measured deflections at the center, when the specimens were tested as columns with flat ends, are shown in Fig. 2. These curves show that the twisting of the specimen or the bending of an outstanding part was more significant, in many cases, than the deflection of the axis of the specimen. It was noted that the shape of the edge of the outstanding plate of Specimen 2A1P-1 changed from double curvature (**S**-curve) to single curvature (**C**-curve) at a load of about 95 000 lb. This change in shape was accompanied by noticeable twisting. Specimen 2A1P-2 showed this same type of action. It is reasonable to believe that the deflections of the outstanding legs of the angles of Specimen 2A1P-3 produced a condition of instability that influenced the strength of the member. The twisting of the pairs of angles of Specimen 4A-2 is probably responsible for the change in the direction of bending of the axis of the specimen; that is, the twisting introduced eccentricities of loading with effects opposite to those of the initial "accidental" eccentricity.

("Accidental eccentricity" is defined as the unintentional eccentricity that must have occurred, including the effects of crookedness of the member, fabrication defects, etc., as well as the error in the location of the point of application of the load.) Specimen 2C-3 also gave a queer curve for the deflections of the north and south sides, which was probably produced by erratic action of the lacing-bar joints. The sudden breaks in the curves of Specimen 2C1P-2 are probably a result of the buckling of the plate.

Analyses of load-deflection data have been developed by Professors W. Ayrton and John Perry,[3] and by Professor R. V. Southwell,[4] by which the eccentricity of loading and the critical column strength of the specimen can be determined. It is then possible to determine the fixation of the ends of the specimen from the critical column strength and the Euler formula for column strengths. There are only a few of these specimens in which the action of the component parts was sufficiently unified and free from twisting that such analyses might have been applied to the data.

Values of accidental eccentricity can be obtained from these data as the eccentricity required to give a good agreement between the measured and computed values of deflection. Such values of accidental eccentricity are not reliable because the measured deflections were greatly influenced by the lack of integral action of the specimen. They would be the upper limiting value of the accidental eccentricity, however. Such values have been used subsequently herein in computing the strength of the members (see heading, "Computed Column Action and Tests to Failure").

The specimens with lacing and relatively light tie-plates show a tendency toward large values of the eccentricity ratio, $\frac{e\,c}{k^2}$, resulting from accidental eccentricities. For the other specimens, the shape of the section seems to have no significant effect on the values of eccentricity ratio resulting from the accidental eccentricity.

DEFLECTION UNDER ECCENTRIC LOADS

The averages of the measured deflections for positive and negative eccentricities agree quite satisfactorily with computed values obtained by the secant formula for deflection:

$$y = e\left(\sec\frac{L}{2\,k}\sqrt{\frac{P}{A\,E}} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In some cases the measured deflection for positive eccentricity is quite different from the value for negative eccentricity. These differences are produced by accidental eccentricities of loading, the magnitude of which can be obtained from curves showing the relation between the deflection and the intentional eccentricity. In Fig. 3, which shows a typical eccentricity-deflection relation, the accidental eccentricity is that which corresponds to zero measured deflection.

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[3] "On Struts," by W. Ayrton and John Perry, *The Engineer* (London), December 10 and 24, 1886, Vol. 62, pp. 464–465, and 514–516.

[4] "On the Analysis of Experimental Observations in Problems of Elastic Stability," by R. V. Southwell, *Proceedings*, Royal Soc. (A) 135 (1932), pp. 601–616; see, also, "Theory of Elasticity," by R. V. Southwell, Oxford Press, 1936, p. 428.

The relations of the measured and computed deflections of the axes of the specimens do not seem to be influenced by the shape of the section. There were a few cases in which the local bending of the component parts was noticeable. These local deflections were influenced by the shape of the section; for example, in the two T-shaped specimens (2A1P-1 and 2A1P-2), the measured values show that the deflection was accompanied by twisting about a longitudinal axis. This action was observed even for small loads.

DEFLECTIONS UNDER OBLIQUE LOADS

In general, the deflections measured at the quarter-points were small. Percentage variations from the computed values are quite large in some cases but the differences between the measured and computed values for the aluminum alloy specimens are, in only one case, greater than 0.008 in. for average stresses of 10 000 lb per sq in. This case is the two-angle, star-shaped specimen (2A-1) for the condition in which the axis of bending is across the tie-plates. Otherwise, the shape of the section seemed to have no significant effect on the relation of the measured and computed deflections of the axes of the specimens. In this case (Specimen 2A-1), the average of the measured values is about 2.5 times as great as the computed value.

The computed values used in these comparisons were obtained as one-eighth the value computed by Equation (2) for the specimen loaded with an eccentricity equal in magnitude to the obliquity. Although this formula does not strictly apply to this type of loading, the error introduced is quite small.

The twisting of the T-shaped specimens was again noticeable even for the low loads and did not change direction with a reversal in the direction of the obliquity.

LACING-BAR STRESSES AND SHEARS

It was found early in the investigation that because of the bending it was necessary to measure the stresses on both faces of the lacing-bars in order to arrive at a satisfactory value of the load on the bar. In some cases the maximum stress in a lacing-bar was more than three times the average stress. With the specimens tested with the condition of flat ends or with eccentric loads, the stresses in the lacing-bars were rather small (less than 4 800 lb per sq in. for average stresses, $\frac{P}{A}$, of 15 000 lb per sq in.). The transverse shears indicated by the average of the lacing-bar stresses were equal to not more than two-thirds of 1% of the total longitudinal load. The transverse shear indicated by the most highly stressed lacing-bar (Specimen 2C-1) was equal to about 1% of the load when the specimen was tested as a column with flat ends. These indicated transverse shears were arrived at as the transverse load required to produce the measured average stress in the lacing-bars, assuming that the bars support all the shear.

With the specimens tested with oblique loads, the stresses in the lacing-bars are somewhat larger and indicate values of transverse shear (based on the average stresses for all the bars of a specimen) as high as 3.9% of the longitudinal load. The typical data in Fig. 4 show the relation between the indicated shear and the ratio of obliquity to the length of the specimen. It should be noticed that the indicated shear, expressed as a percentage of the total load, is independent of the load and can be represented by an empirical equation of the form,

$$V_i = F + G\left(\frac{2\,q}{100\,L}\right) \ldots\ldots\ldots\ldots (3)$$

The data indicate the maximum values of the constants, F and G, to be 0.7 and 1.7, respectively. It is shown in Fig. 4 that the relation between the indicated shear and the obliquity is a straight line, and that for individual lacing-bars there is some indicated shear even for zero intentional obliquity.

The average stresses in the lacing of the specimens with heavy batten-plates at the ends (4A-1 and 4A-2) indicate transverse shears between one-third and two-thirds of the transverse component of the load. The specimens with lighter batten-plates at the ends have an indicated shear between 0.8 and 1.7 times the transverse component of the load. The average stresses in the lacing of the specimens with one cover-plate and one plane of lacing indicate a transverse shear on the lacing equal to about one-fourth the transverse component of the load.

Some of the equations commonly used for estimating the shear in built-up columns follow from analyses of the shape of the deflected structure. In generalizing and reducing the formulas, the value of the modulus of elasticity is taken as the initial modulus and the value of the average stress is taken as the ultimate strength of the member.[5] For use with structural steel (which has a very definite yield strength) this combination of values might be acceptable, but with the high-strength steels and non-ferrous metals which do not have such a definite point of yielding this is subject to criticism. An effective modulus of elasticity for strengths above the elastic stress range might be used in the generalization but the resulting formulas would be complicated considerably.

Other equations in common use are based on analyses that consider the column as a beam and the shear load producing a bending stress equal to the reduction in strength given by the slenderness-ratio term of the straight line or Rankine formulas.[6] When used to estimate the shear at failure, these analyses are subject to the foregoing criticism.

The data obtained on these specimens indicate that, for average stresses within the elastic range, the transverse shear on axially loaded columns is a constant percentage of the load on the specimen. In the case of Specimen 2C-1 (with the highest indicated shear), the transverse shear at failure would be about 2 350 lb if the relation between shear and axial load were the same (1%) at failure as at working stresses. The average stress in the most highly stressed lacing-bar would be 4 930 lb per sq in., whereas the Euler column strength of the bars is 7 400 lb per sq in. Since the failure of the specimen (described subsequently) was accompanied by the buckling of the lacing-bars, either the shear was greater than 1% of the load or the bar was in effect an eccentrically loaded column. Either explanation is tenable. This shear of 2 350 lb represents a transverse load of 330 lb per sq in. of cross-section of the main members. Although the results of this investigation do not definitely establish a value for the transverse load, they indicate that the value of 600 lb per sq in. of cross-section used in the old A. R. E. A. (American Railway Engineering Association) specifications for pin-ended columns is quite reasonable. These test data also indicate that the shear on eccentrically loaded columns should be taken greater than 1%, the amount depending on the value of the eccentricity ratio, $\frac{e\,c}{k^2}$.

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[5] Rept. of Committee XV on Iron and Steel Structures, *Bulletin No. 374*, Am. Ry. Eng. Assoc., February, 1935; see, also, "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 422.

[6] "Structural Engineers' Handbook," by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., 1924, p. 733.

Formulas for Predicting Column Strengths

Before proceeding with the discussion of the tests, a few remarks will be interjected concerning formulas for predicting the column strength and the type of failure that might be expected.

The straight-line type formula is commonly used for predicting the strength of columns that fail by the sidewise bending of the member as a unit. This equation is of the form,

$$\frac{P}{A} = s_0 - C\frac{K\,L}{k} \quad \text{.......................} (4)$$

in which s_0 = column strength, in pounds per square inch, corresponding to zero slenderness ratio. The range of slenderness ratios is limited for which this equation is applicable; but the value of C can be chosen so that the combination of this equation with the Euler column formula covers the full range of slenderness ratios. This combination of formulas has been found applicable to the various aluminum alloys.[7]

Equations for stresses in columns, such as Equation (1*a*) and that of Professors Ayrton and Perry,[8]

$$s = \frac{P}{A}\left(1 + \frac{P'}{P' - P}\frac{e\,c}{k^2}\right) \quad \text{....................} (5)$$

can be used also for predicting the ultimate strength of columns that fail by sidewise bending as a unit. The value of s in these equations is set equal to the compressive strength of the material and a value of Load P is determined that satisfies the equation. The solution is usually most easily obtained by trial. It will be noticed that the value of the modulus of elasticity enters each of these equations, appearing indirectly in Equation (5). This presents no particular problem in the case of structural steel, which has a definite yield strength; but for some of the high-strength steels and non-ferrous metals the evaluation of an effective modulus of elasticity [9] must be made.

Although Equations (1*b*), (4), and (5) can be used to predict the ultimate strength of a column that fails by sidewise bending of the member as a unit, if the member is subject to failure by local buckling or twisting, it must be treated as a local instability problem. Naturally, the strength of the member will be limited by the lowest of the strengths predicted by a consideration of all the types of failure that might occur. Equations have been derived for some special cases of local buckling.

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[7] "Column Strength of Various Aluminum Alloys," *Technical Paper No. 1*, Aluminum Research Laboratories, Aluminum Co. of America, 1938.

[8] "On Struts," by W. Ayrton and John Perry, *The Engineer* (London), December 10 and 24, 1886, Vol. 62, Equation (22); also "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am Soc. C. E., Vol. 101 (1936), p. 422.

[9] "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 467.

For the case of wide thin plates, the critical stress, s_{cr}, at which buckling occurs, can be computed by the equation:[10]

$$s_{cr} = \frac{a\,E}{1 - \mu^2}\left(\frac{t}{b}\right)^2 \quad\text{.........................(6)}$$

in which a = a coefficient that depends on the proportions of the plate and the type of restraint along the edges of a plate; μ = Poisson's ratio; t = thickness; and b = width of plate. Values of Coefficient a for various loading conditions and various conditions of the edges of the plate can be obtained from the reference giving the equation. It should be borne in mind that this value of s_{cr} is the maximum average stress that can be developed in the plate and at which buckling occurs. The agreement between the critical stress and the average stress at which collapse occurs will depend on the shape and proportions of the entire cross-section of the column and the loading conditions of the member; that is, in the specimens of which the plate is the principal part of the member, the agreement between the critical stress and the average stress at failure should be rather close, whereas for a specimen in which the plate is merely an appendage, the average stress at final collapse of the entire member may be considerably greater than the critical stress at which the plate buckles. The average stress at the collapse of a specimen with round or pinned ends will probably agree very well with the computed critical stress and, of course, will be less than the test result for a similar specimen with restrained or fixed ends.

Some cases of local buckling can be treated satisfactorily by using a proper value of s in Equations (1*b*) and (5). These formulas would include those cases in which the maximum stress, due to combined axial load and bending, is developed in some part that is subject to local buckling. The value of s then is the compressive strength of that part and can probably be determined most satisfactorily by a compression test on a short length of the part.

COMPUTED COLUMN ACTION AND TESTS TO FAILURE

The computed strengths of the specimens and the types of failure to be expected are given in Table 3. In Column (2), the larger of the two numbers corresponds to the fixed-edge condition and the smaller to the simply supported edge condition. In Column (4), Table 3, two values are listed for Specimens 2C-1 and 2C-2. The first value corresponds to the case of bending about the axis of least computed stiffness and the second to the case of bending about the axis of greatest computed stiffness. The difference in the values of accidental eccentricity for the two directions makes this investigation of strength necessary. In calculations involving stresses above the elastic range, an effective modulus of elasticity has been used. The effective modulus was assumed to

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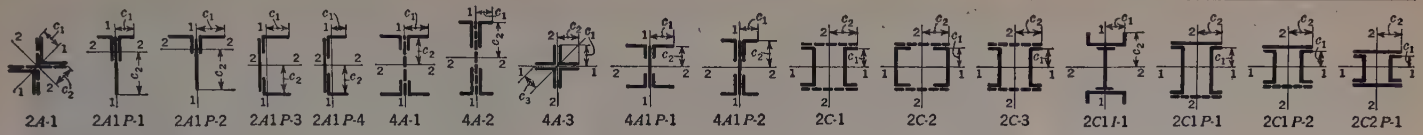
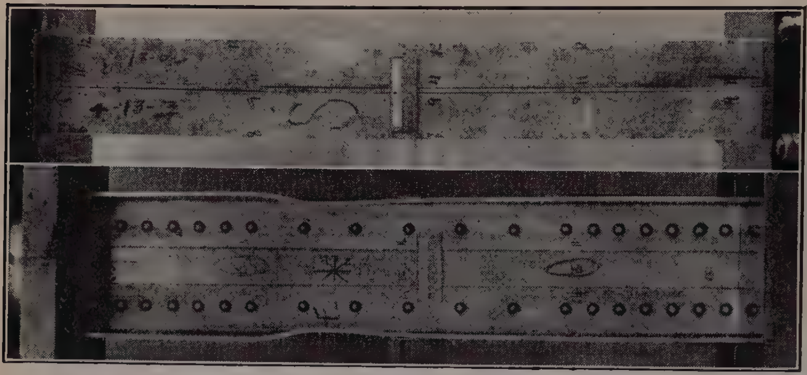
[10] "Strength of Materials, Part II," by S. Timoshenko, Van Nostrand, 1930, p. 605; see, also, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1009; and "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill, 1936, Chapter VII.

TABLE 3.—Computed Strengths and Test Results; Built-Up Members Tested as Columns with Flat Ends

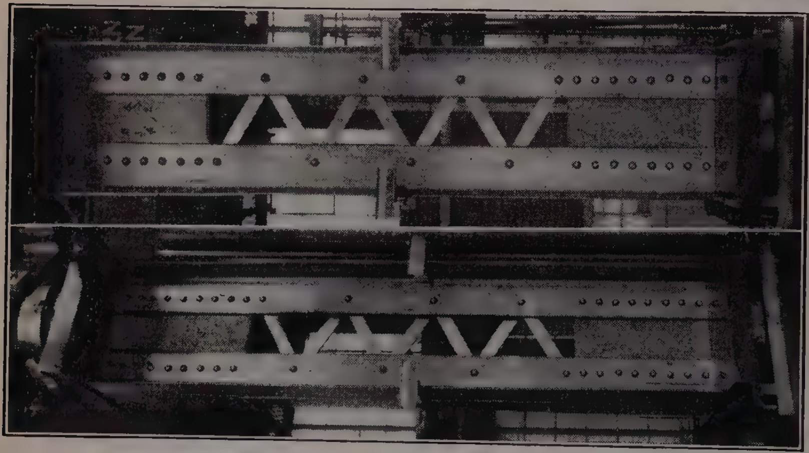


Specimen No. (see Table 1)	Strengths, in Pounds per Square Inch			Type of failure predicted	Maximum load, P, in pounds	Column strength, $\frac{P}{A}$, in pounds per square inch	Remarks
	Critical strength by Equation (6)*	Column Strengths by: Equation (3)	Column Strengths by: Equation (5)				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2A-1	25 600	39 400	34 500	Twisting of full length*	179 500	25 460	....
2A1P-1	13 850 5 200	34 300	35 000	Twisting of full length*	156 400	20 390	Stress at the "break" in the load-deflection curve = 12 500
2A1P-2	12 400 4 700	34 900	35 200	Twisting of full length*	156 500	20 400	Stress at the "break" in the load-deflection curve = 7 000
2A1P-3	27 800 17 800	33 500	33 900	Buckling of outstanding legs*	211 800	27 580	....
2A1P-4	30 900 26 250	33 600	32 700	Buckling of outstanding legs	222 800	29 000	Failure accompanied by buckling of the long legs of the angles and of the plate
4A-1	29 000 17 800	33 500	29 400	Buckling of outstanding legs*	185 000	27 380	....
4A-2	30 900 27 100	33 000	31 800	Buckling of angles	219 800	32 230	Critical strengths of long legs of angles between 29 300 and 29 900
4A-3	31 400 29 300	31 900	29 700	Twisting of full length*	252 500	32 080	If the angles should act individually so $\frac{b}{t} = 16$, the critical strength is between 17 800 and 28 100
4A1P-1	31 500 25 600	34 200	34 600	Buckling of outstanding legs	249 200	35 000	Failure accompanied by sidewise bending
4A1P-2	32 900 28 000	33 800	33 400	Buckling of outstanding legs	252 500	35 460	Failure accompanied by sidewise bending
2C-1	....	32 100	32 700 29 000	Bending about axis parallel to the web of the channels*	235 000	32 600	....
2C-2	....	31 400	33 300 33 800	Bending about axis parallel to the planes of lacing*	249 500	34 800	....
2C-3	....	40 800	39 400	Bending about axis parallel to the planes of lacing*	273 000	38 340	....
2C1I-1	....	39 300	41 000	Bending about axis parallel to the web of the I-beam*	290 500	40 600	....
2C1P-1	26 100 18 000	34 200	35 400	Buckling of the plates*	251 400	31 820	The plate was noticeably wrinkled at a stress of 29 000
2C1P-2	37 200	37 200	36 500	Bending about axis parallel to the plate*	251 400	33 200	....
2C2P-1	29 800 17 000	38 200	39 900	Buckling of the plates*	247 100	31 400	....

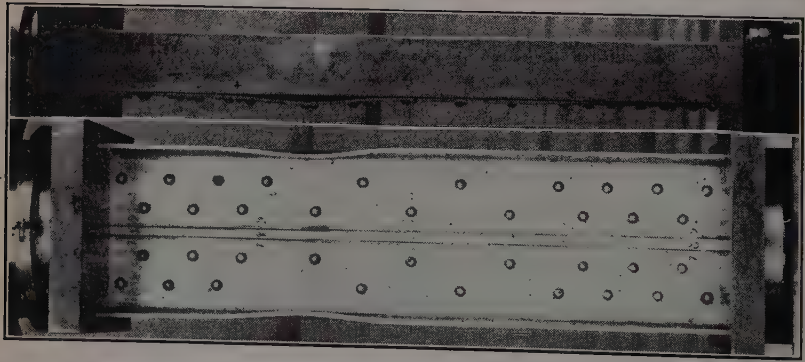
* Prediction fulfilled.



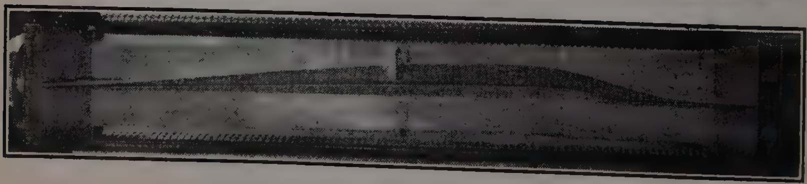
4A1P-1



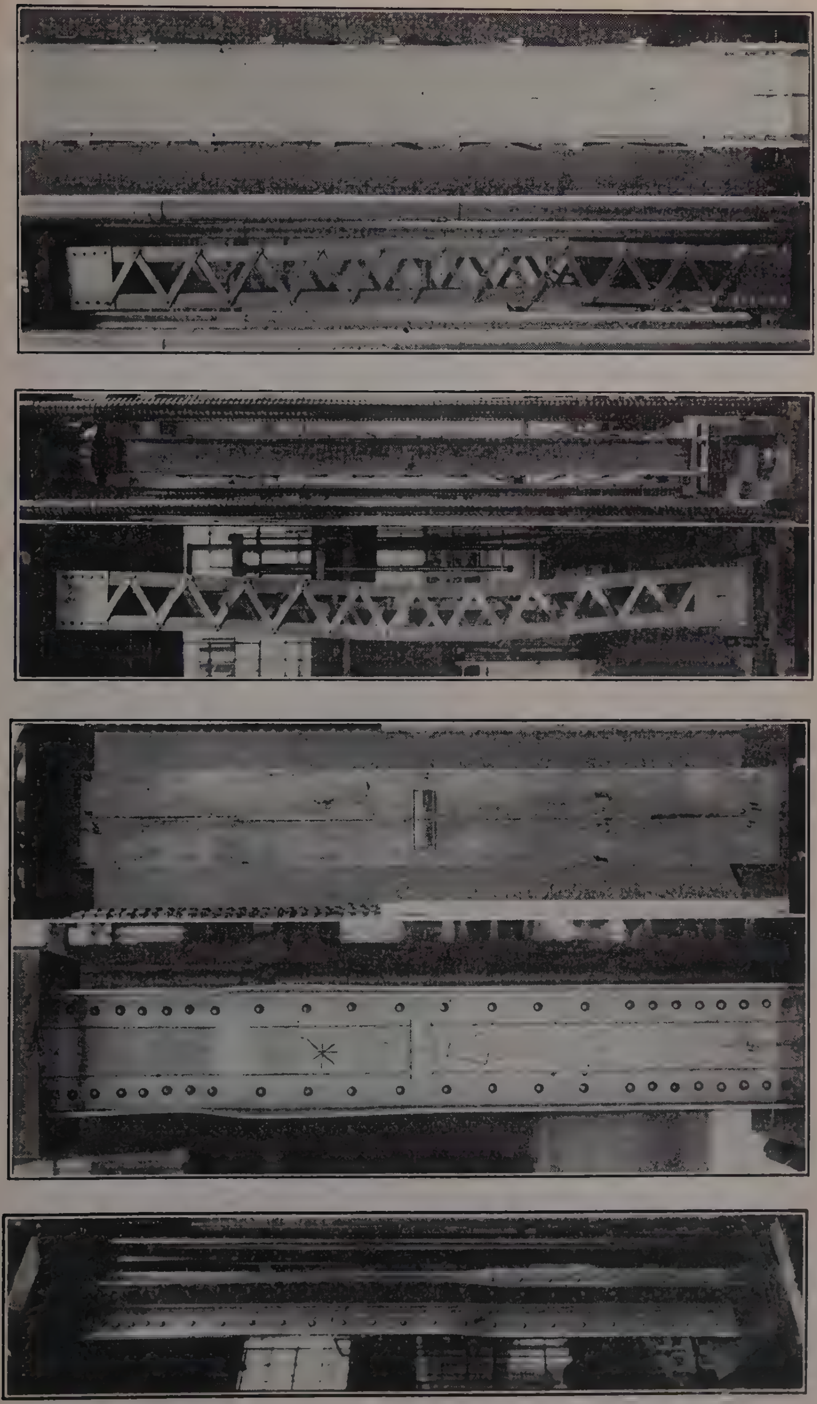
4A-2



2A1P-4



2A1P-2



2C-3

2C-1

4A1P-2

4A-3

Fig. 5.—Photographs of Selected Specimens, During and After Tests

decrease along a straight line from the initial value at the upper limit of the elastic range to the value of the tangent modulus at the yield strength (0.2% set) and from this decreased value along another straight line to zero at the maximum stress.

The column strength of the member acting as a unit was obtained by the straight-line formula, Equation (4), based on the properties of the material in the specimen and also by Equation (5), using a value of accidental eccentricity consistent with the data in Fig. 2. In these computations, the value of K was taken equal to 0.5 which assumes that the specimens acted as columns with fixed ends. For each specimen the critical strength based on the local buckling of the plates was obtained by Equation (6) using values of the coefficient corresponding to two widely different conditions of edge support, namely, simply supported edges and fixed edges. The actual edge conditions usually provide some restraint, so the critical strength that can reasonably be expected can be obtained by interpolation between these extreme values. The predicted type of failure is that corresponding to the lowest computed strength, whether by local buckling or by bending as a unit. The maximum loads and column strengths are for these specimens tested as columns with flat ends. In nearly all cases the type of failure was predicted correctly. In a few cases the failure appears to be a combination of buckling and bending or of buckling of more than one part.

The strength of Specimen 2A-1 was computed on the basis of simple support along the back of the plate. This seems reasonable because the four outstanding legs are all equally subject to buckling with the resultant twists centering about the shear centers of the individual angles. This reasoning does not apply to Specimen 4A-3 (Fig. 5), because, although the four legs are all equally subject to buckling, the resultant twist must center about the heels of the individual angles. Thus, the angles are stronger than similar angles, free to twist about their own shear centers. The outstanding plates of the other specimens are all restrained to a certain extent. The two values of computed critical stress given in Table 3 are based on the two conditions of the back edge of the plate mentioned previously. For those specimens with computed strengths in the plastic range the effect of the restraint at the edge of the plate is not so pronounced as in the cases in which the strength is in the elastic range. This results from the fact that the effective modulus of elasticity varies with the stress.

A study of the load-deflection curve and the stresses measured at the free edge of the plate of Specimen 2A1P-1 indicates that the value of computed critical stress corresponding to fixed edges is close to the average stress at which buckling occurred, whereas a similar study for Specimen 2A1P-2 (Fig. 5), indicates that the value corresponding to fixed edges is much too great. The differences between the ultimate strengths and these computed critical strengths should not be surprising in view of the definitions of terms and the previous discussion of Equation (6). The ultimate strength of Specimen 2A1P-4 (Fig. 5) was probably reduced somewhat by the buckling of the plate.

The critical strength of Specimen 4A-2 is computed for the outstanding (short) legs of the angles with a ratio of width to thickness of 12 and a large ratio of length to width. It is also computed for the inside (long) legs with a ratio of width to thickness of 16 and a length equal to the width. The range of values based on both simple and built-in support at the back edge of the plate is from 27 000 lb per sq in. to 30 900 lb per sq in. The reversal of the load-deflection curve in Fig. 2(*b*) at a stress of about 26 000 lb per sq in., is probably associated with the buckling of the legs of the angles. The ultimate strength is 32 230 lb per sq in.

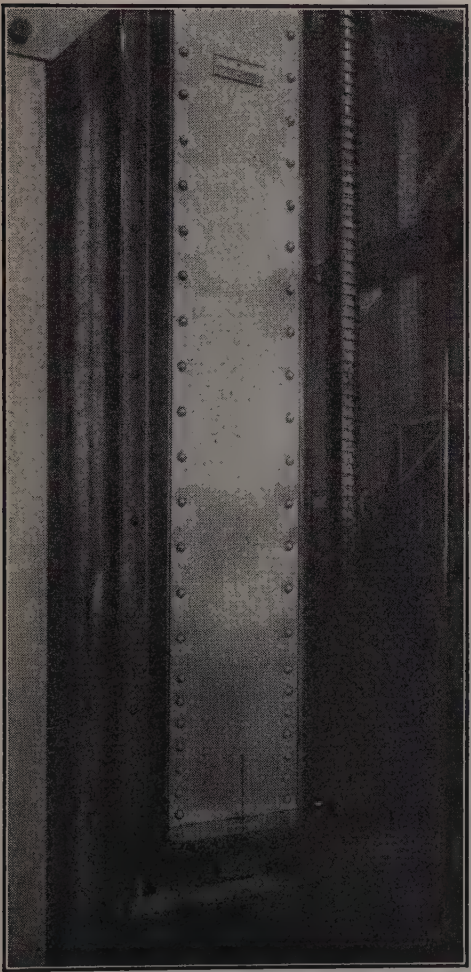


FIG. 6.—VIEW OF SPECIMEN 2C1P-1, WITH AN AVERAGE STRESS OF 29 000 POUNDS PER SQUARE INCH

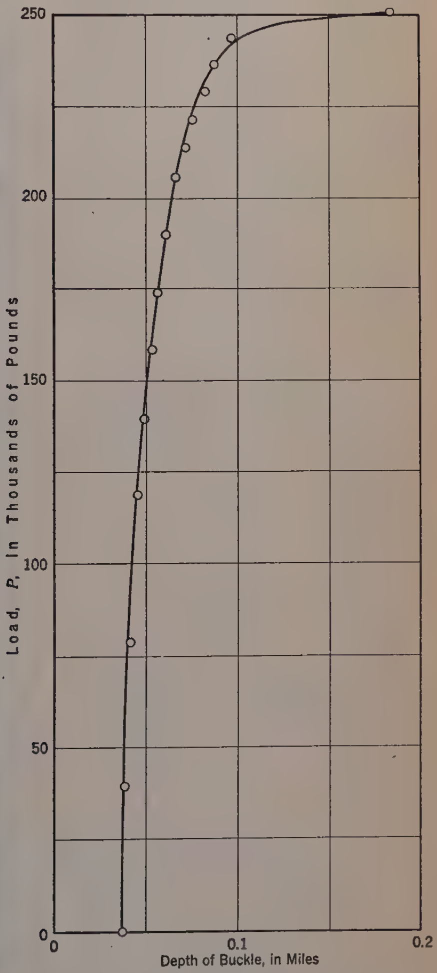


FIG. 7.—RELATION BETWEEN THE LOAD AND THE DEPTH OF BUCKLE, IN PLATE OF SPECIMEN 2C1P-2

Specimens 4A-3, 4A1P-1, and 4A1P-2 (Fig. 5), with outstanding legs of angles, having width-to-thickness ratios of 8, 12, and 10, respectively, developed strengths equal to the values computed by the straight-line formula and by

Equation (5). Specimen 4A-2 (Fig. 5) developed 98% of the strength predicted by the straight-line formula, and Specimens 2A1P-1 and 2A1P-2 developed only about 60% of the strength similarly computed. These comparisons should emphasize the fact that short columns with outstanding plates having ratios of width to thickness greater than about 12 are subject to local failure at loads less than those predicted by the usual column formulas.

Specimens 2C-1 and 2C-3 (Fig. 5) can scarcely be said to act as units in view of the fact that failure occurred by bending about the axis of greatest computed stiffness. The ratio of moments of inertia is 1 : 1.3. The failure of Specimen 2C-3 (Fig. 5) was accompanied by the formation of Lueders' lines. The agreement between the test results and the computed values was very good.

Fig. 6 shows Specimen 2C1P-1 under an average stress of 29 000 lb per sq in. The buckling of the plate is quite apparent although at an average stress of 28 000 lb per sq in., the plate appeared smooth. These values agree fairly well with the critical strength computed on the basis that the edges of the plate are built in and the width is the clear distance between the rows of rivets.

Fig. 7 shows the relation between the load and the depth of a buckle in the plate of Specimen 2C1P-2. The plate buckled rather suddenly at a load of 205 000 lb. This load coincides with that at which the "break" occurred in the load-deflection curve of Fig. 2(*b*). Besides the formation of this buckle, failure was accompanied by the formation of Lueders' lines. The computed strengths for this specimen given in Table 3 are about equal to the yield strength of the material. The test result is about 10% less than this, probably as a result of the initial unevenness of the plate.

The computed critical strength of Specimen 2C2P-1 agrees fairly well with the break in the load-deflection curve. The ultimate strength exceeds this value by 19 per cent. None of the three specimens, with a ratio of width of plate (between the lines of rivets) to thickness of about 46, developed the yield strength of the materials nor the strength predicted by the straight-line formulas.

CONCLUSIONS

The following conclusions regarding built-up columns of high-strength aluminum alloys are justified by the foregoing data and discussion:

(1) For low loads the shape of the cross-section has no significant effect on the distribution of stress; but for the higher loads the specimens with thin outstanding plates have a pronounced variation in the stress;

(2) The shape and proportions of the cross-section have a significant effect on the ultimate strength and the type of failure, whether by bending as a unit or by local buckling;

(3) The type of failure, whether by local buckling of a component part or by bending as a unit, can be predicted as the type of failure corresponding to the least of the strengths computed by the various equations for critical stress and column strength;

(4) In order to avoid failure of short columns $\left(\frac{L}{k} = \text{about } 40\right)$ by local buckling at a load less than that predicted for the column acting as a unit, the

width-to-thickness ratio of outstanding plates and legs of angles should not exceed about 12 and the ratio of width to thickness of cover-plates with a row of rivets near each edge should not exceed about 40;

(5) Local buckling will not always result in the collapse of the entire member, but may be only a wrinkling of the buckled part;

(6) In the range of working loads the shape and proportions of the cross-section have no significant effect on either the ratio of measured deflection of the axis of the member to be the computed deflection or on the magnitude of the indicated shear, expressed as a percentage of the load; and,

(7) For two-channel members with two planes of lacing to develop equal column strengths in the two principal directions, the moment of inertia of the section parallel to the lacing, computed in the usual manner, should be greater than that parallel to the webs of the channels in order to counteract the minute unintentional slack in the lacing-bar joints.

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The writer wishes to acknowledge the assistance of the following members of the staff of the Physical Testing Division of the Aluminum Research Laboratories: R. L. Templin, M. Am. Soc. C. E., Chief Engineer of Tests; R. G. Sturm, Assoc. M. Am. Soc. C. E., Research Engineer Physicist; and E. C. Hartmann, Assoc. M. Am. Soc. C. E., Research Engineer.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[11] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

A = area of cross-section, main members, in square inches;
a = a coefficient in Equation (3), that depends on the proportions of the plate and the type of restraint along the edges of the plate;
b = breadth; width of plate, in inches;
C = a coefficient chosen so that the combination of Equation (4) with the Euler formula covers the full range of slenderness ratios;
c = distances, in inches, from the neutral axis to the point where the stress is desired;
E = modulus of elasticity of the material;
e = eccentricity of loading, in inches;
F = term in Equation (3);
G = term in Equation (3);
K = a coefficient describing the end conditions; taken as unity for a column with round ends, and as 0.5 for a column with fixed ends;

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[11] A.S.A.—*Z10A*—1932.

k = radius of gyration of the cross-section, in inches, assuming unified action of the component parts of the member;

L = length, in inches;

P = total load, in pounds; P' = Euler column load, in pounds;

q = obliquity of loading, in inches; distance between the axis of the columns at the ends, and the plane of the knife-edges;

s = average unit stress, in pounds per square inch $= \frac{P}{A}$; s_{cr} = critical buckling stress; s' = Euler's column strength $= \frac{\pi^2 E}{\left(\frac{K L}{k}\right)^2}$; s_0 = column strength corresponding to zero slenderness ratio;

t = thickness, in inches;

V = total shear, in pounds; V_i = indicated shear, expressed as a percentage of the total load, P;

y = deflection, in inches; and,

μ = Poisson's ratio.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

DESIGN OF DOWELS IN TRANSVERSE JOINTS OF CONCRETE PAVEMENTS

By Bengt F. Friberg,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Technical literature lacks a rational solution for the deflections and stresses in and around dowels through transverse joints in concrete pavements. The action of the dowel may be established according to general elastic formulas governing for an elastic structure in a yielding mass. These elastic equations are presented herein in simple form and are applied to round dowels such as may be used in road joints under assumed, approximately correct, conditions of yield in the concrete around the dowels. The economical length of dowels is indicated.

A reasonable rule for the rapid appraisal of the effect of dowel spacing is stated and applied to varying dowel spacing. The concrete stress relief furnished by dowels, being a function of dowel deflection and pavement deflection, has been established in practical form based upon the theoretical deflection conditions. The use in design is indicated, and experimental values of stress relief are checked with those obtained with this method of design.

Knowledge of deflections in and around dowels makes it possible to estimate the loads and stresses resulting from inaccurate dowel installation. The values have been developed and applied to experiments of record, giving some indication of the strength of the concrete. The variation in dowel alignment that may be permitted in actual construction, can be gaged to varying joint widths, movements, and dowel sizes.

Present Construction Practice

Transverse joints in concrete pavements are necessary to make longitudinal movements of the pavement possible, and to relieve shrinkage and temperature stresses. In most highway designs round steel dowels are placed across these transverse joints, consisting of $\frac{3}{4}$-in. or $\frac{7}{8}$-in. round plain bars, 2 ft long, spaced

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January 15, 1939.**

[1] Research Engr., Laclede Steel Co., St. Louis, Mo.

from 12 in. to 20 in. on centers. One end of the dowel is capped and painted or greased to present the least possible resistance to movement of the pavement. The dowels are most frequently in direct contact with the surrounding concrete. In some designs, however, they project into closely fitted heavy sleeves, which, in turn, bear in the concrete. Practical construction considerations, observation of performance, and experimental data rather than design analysis, have led to the construction practices in common use.

The dowels should be required to keep the road surfaces on each side of the joint in alignment even if subgrade support is lacking. They are also required to relieve the concrete stresses due to wheel loads in the immediate vicinity of the joints by transferring a part of these wheel loads to the adjacent pavement. To perform these functions the dowels themselves must be strong enough to carry a reasonable part of load across the joint; and, further, they must be dimensioned so that the bearing pressure between the dowel and the concrete surrounding it does not exceed permissible values. They must transfer their portion of the wheel loads with a sufficiently small deflection to offer a measure of side support to the loaded side of the joint even if the pavement rests on a relatively non-yielding subgrade.

A general mathematical solution of the design problem with specific application to dowels through joints has not appeared in technical literature, although the methods have been referred to by investigators and solutions have been provided for application in specific cases, or under assumed distribution of concrete pressure along the dowel.[2,3] The exact mathematical solution of the problem pertaining to a dowel structure of infinite length has been presented by Professor S. Timoshenko and Mr. J. M. Lessels.[4]

As the deflection of a long dowel in concrete, and the resultant pressure distribution between the dowel and the concrete, are not difficult to establish mathematically, these solutions are presented in the following analysis and from them the dowel performance and dowel effectiveness under common construction conditions have been determined. It has been necessary to use material constants with regard to the deflection of the dowel in the concrete as well as with regard to the deflection of the pavement on the subgrade, which have not been experimentally determined with accuracy. Numerical values included in the paper, therefore, are not intended to be used other than for illustration of the method presented, or with respect to specific experimental evidence.

Notation.—The letter symbols in this paper are defined where they first appear and are arranged, for convenient reference, in the Appendix.

General Solution

For an elastic structure (see Fig. 1) extending an infinite distance into an elastic mass from its boundary surface, Professor Timoshenko has developed the relationship between the elastic properties of the two structures and the

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[2] "Design of the Reinforced Concrete Road Slab," by L. E. Grinter, M. Am. Soc. C. E., *Bulletin No. 39*, Texas Eng. Experiment Station.

[3] "Design of Joints in Concrete Pavements," by R. D. Bradbury, Assoc. M. Am. Soc. C. E., *Proceedings*, Twelfth Annual Meeting, Highway Research Board, 1932.

[4] "Applied Elasticity," by S. Timoshenko and J. M. Lessels, Westinghouse Technical Night School Press, pp. 133–141.

deflection in the surrounding mass. Fig. 1 is a loading and deflection diagram that illustrates the sign convention adopted.

The relative stiffness of the structure and the mass is expressed by the following constant:

$$\beta = \sqrt[4]{\frac{K\,b}{4\,E_s\,I}} \qquad (1)$$

in which K = a "modulus of support" in an elastic mass, in pounds per inch³; b = the width of the structure; E_s = the modulus of elasticity of the structure (28 000 000 lb per sq in.); and I = the moment of inertia of the structure.

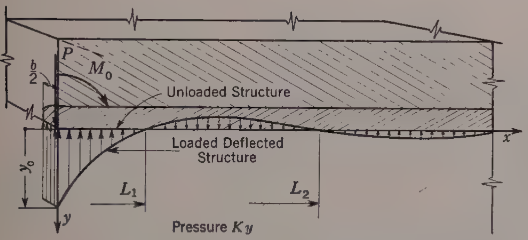

FIG. 1.—LOAD AND DEFLECTION DIAGRAM, SHOWING AN ELASTIC STRUCTURE EMBEDDED IN AN ELASTIC MASS

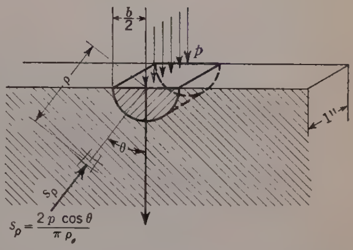

FIG. 2.—DISTRIBUTION OF STRESS IN AN ELASTIC MASS UNDER A DOWEL IF THE CONCRETE ABOVE THE DOWEL IS NEGLECTED

Modulus K is defined as the pressure intensity, in pounds per square inch, on the elastic mass, required to cause a 1-in. settlement. As developed by Professor Timoshenko, the general equation for the deflection, y, of the structure, in the mass, is:

$$y = \frac{e^{-\beta x}}{2\,\beta^3\,E_s\,I}\left\{P\cos\beta\,x - \beta\,M_0\,(\cos\beta\,x - \sin\beta\,x)\right\} \qquad (2)$$

in which e = the base of Naperian logarithms; x = distances from the left end of the structure; P = the concentrated load, in pounds, acting downward at the face of the elastic mass; and M_0 = the bending moment in the structure at the left face of the elastic mass, in inch-pounds. Moment M_0 is positive clockwise. The slope of the structure may be established as the first derivative of Equation (2):

$$\frac{dy}{dx} = \frac{e^{-\beta x}}{2\,\beta^2\,E_s\,I}\left\{(2\,\beta\,M_0 - P)\cos\beta\,x - P\sin\beta\,x\right\} \qquad (3)$$

For x equal to 0, the deflection, y_0, at the face of the mass is:

$$y_0 = \frac{P - \beta\,M_0}{2\,\beta^3\,E_s\,I} \qquad (4)$$

The pressure between the structure and the mass at any point is equal to $K\,y$ and, therefore, is derived directly from the deflection. The points where the pressure on the structure changes from positive to negative are obtained from Equation (2) for $y = 0$; thus:

$$\tan \beta L = 1 - \frac{P}{\beta M_0} \qquad (5)$$

in which L is the distance from the face of the mass to these points. The moment, M, at any point on the structure is obtained directly from Equations (2) and (3):

$$- E_s I \frac{d^2 y}{dx^2} = M = - \frac{e^{-\beta x}}{\beta} \{P \sin \beta x - \beta M_0 (\sin \beta x + \cos \beta x)\} \qquad (6)$$

From this value of M, the shear in the structure, V, at any point may be computed directly:

$$\frac{dM}{dx} = V = - e^{-\beta x} \{(2 \beta M_0 - P) \sin \beta x + P \cos \beta x\} \qquad (7)$$

APPLICATION OF DESIGN FORMULAS

The assumption is made that the dowels are not bonded to the concrete and that they fit snugly into the concrete mass. Under these assumptions it is evident that Equations (1) to (7) are directly applicable to long dowels. Load P is equivalent to the shear acting on the dowel, transferred from one road slab to the next. Moment M_0 is the moment in the dowel at the face of the joint incidental to the transfer of this shear. In practice, the dowels are usually not very stiff compared with the concrete pavement. It is an acceptable approximation, then, to presume that a point of contraflexure exists in the dowel at the center of the joint, in which case, for a joint width, a:

$$M_0 = - P \frac{a}{2} \qquad (8)$$

Formulas for the solution of long dowels may then be written in the following simple and usable forms: From Equation (4) the deflection of the dowel with respect to the concrete, at the face of the joint, is:

$$y_0 = \frac{P}{4 \beta^3 E_s I} (2 + \beta a) \qquad (9)$$

From Equation (3) the slope of the dowel at the face of the joint is,

$$\frac{dy_0}{dx} = - \frac{P}{2 \beta^2 E_s I} (1 + \beta a) \qquad (10)$$

From Equation (5) the points, L_n, where the pressure changes direction along the dowel, are:

$$\tan \beta L = \frac{2 + \beta a}{\beta a} \qquad (11)$$

Equation (7), for $\frac{dM}{dx} = 0$, determines the point, x_m, where maximum moment occurs in the dowel:

$$\tan \beta \cdot x_m = \frac{1}{1 + \beta a} \qquad (12)$$

By substituting this value in Equation (6), the maximum moment in the dowel is obtained:

$$M_{\max} = -\frac{P e^{-\beta x_m}}{2 \beta} \sqrt{1 + (1 + \beta a)^2} \qquad (13)$$

The stresses and deflections are proportional to P. The locations of the point of maximum moment and of the dowel action, as represented by L_n, are independent of the magnitude of P.

YIELDING OF CONCRETE IN WHICH DOWELS ARE EMBEDDED

Values of K in Equation (1) have not been determined experimentally. The only values known to the writer are those suggested[2, 5] by L. E. Grinter, M. Am. Soc. C. E., of 300 000 lb per in.^{3} and 1 500 000 lb per in.^{3}. K represents the elastic deformation of several inches of concrete above or below the dowel; and its numerical value, therefore, would be less than the modulus of elasticity of the concrete.

The dowels entering a concrete pavement slab are completely surrounded by concrete. The concrete above the dowel holds that immediately beneath the dowel by tension. The compressive stress in the concrete must then be less than the value existing at the dowel in reverse proportion to the depth beneath it. The stress condition illustrated in Fig. 2 is theoretically correct for loadings at a boundary surface. In actual road construction the vertical depth of the concrete beneath the dowel is usually from 3 in. to 6 in., and the vertical pressure in the concrete at the bottom of the slab cannot be greater than the bearing pressure exerted by the slab on the soil. It follows, therefore, that for the average grade of concrete now in use, the value of K probably is higher than 300 000 lb per in.^{3}. In this paper K has been assumed equal to 1 000 000 lb per in.^{3} for all sizes of dowels.

Since the value of K is dependent, to a major degree, upon the pressure immediately at the dowel, its value will probably increase with an increase in the size of dowels; and the experimental establishment of the value, $K\,b$, is highly desirable. However, since β varies as $K^{\frac{1}{4}}$ (see Equation (1)), a considerable change in K will not influence the stress distribution to any great extent.

DESIGN FORMULAS FOR THE ORDINARY CASE

All the design formulas may be solved directly. Table 1 affords a clear "picture" of the stresses in and around dowels. The maximum moment in the dowels is seen to occur some distance away from the joint, inside the concrete.

For shears other than 1 000 lb, the values of deflections and stresses change in direct proportion for values within the yield point of the steel or the ultimate bearing resistance of the concrete. Stresses above the yield point in the steel

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[5] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1157.

(first reached at the point of maximum moment inside the concrete) have the effect of increasing the maximum bearing pressure on the concrete at the face of the joint. Under most construction conditions the bearing stress on the concrete is the critical one. Accordingly, the steel in the dowels should have a sufficiently high yield point so that it is not reached before the bearing stress on the concrete has exceeded any possible ultimate value.

TABLE 1.—STRESS CHARACTERISTICS OF DOWELS FOR A SHEAR OF 1 000 POUNDS; $K = 1\,000\,000$

Width of joint, a, in inches	Deflection, y_0, in inches, at the face of the joint	Bearing stress, $K y_0$, in pounds per square inch, imposed on the concrete at the face of the joint	DISTANCES, IN INCHES, FROM THE FACE OF THE JOINT TO THE POINT OF:			Maximum moment, in inch-pounds	Deflection, y_0, in inches, at the face of the joint	Bearing stress, $K y_0$, in pounds per square inch, imposed on the concrete at the face of the joint	DISTANCES, IN INCHES, FROM THE FACE OF THE JOINT TO THE POINT OF:			Maximum moment, in inch-pounds
			First stress reversal, L_1	Second stress reversal, L_2	Maximum moment, x_m				First stress reversal, L_1	Second stress reversal, L_2	Maximum moment, x_m	
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	(a) ¾-INCH DOWELS; $I = 0.016$ INCH^4; $\beta = 0.810$ INCH^{-1}						(b) ⅞-INCH DOWELS; $I = 0.029$ INCH^4; $\beta = 0.723$ INCH^{-1}					
0	0.0022	2 160	1.94	5.81	0.97	400	0.0017	1 650	....	....	....	....
0.25	0.0024	2 380	1.82	5.70	0.84	490	0.0018	1 800	....	....	....	....
0.50	0.0026	2 590	1.73	5.60	0.76	575	0.0019	1 950	....	....	....	....
0.75	0.0028	2 810	1.65	5.53	0.69	670	0.0021	2 009	....	....	....	....
1.00	0.0030	3 030	1.59	5.46	0.62	770	0.0022	2 240	....	....	....	....
1.50	0.0035	3 460	1.49	5.36	0.53	980	0.0025	2 540	....	....	....	....
	(c) 1-INCH DOWELS; $I = 0.049$ INCH^4; $\beta = 0.654$ INCH^{-1}						(d) 1¼-INCH DOWELS; $I = 0.120$ INCH^4; $\beta = 0.552$ INCH^{-1}					
0	0.0013	1 300	2.40	7.20	1.20	495	0.0009	890	2.84	8.52	1.43	585
0.25	0.0014	1 410	2.28	7.08	1.07	575	0.00095	950	2.72	8.40	1.31	665
0.50	0.0015	1 520	2.19	7.00	0.99	665	0.0010	1 010	2.62	8.30	1.21	755
0.75	0.0016	1 620	2.10	6.90	0.90	760	0.0011	1 070	2.54	8.20	1.12	850
1.00	0.0017	1 730	2.03	6.84	0.83	860	0.0011	1 130	2.46	8.15	1.04	945
1.50	0.0019	1 940	1.91	6.71	0.72	1 060	0.0013	1 250	2.32	8.00	0.91	1 145

LENGTH OF DOWELS

The foregoing theoretical analysis of dowels, of course, is correct only for an infinitely long dowel. In Fig. 3, the typical diagrams for concrete pressure, dowel shear, and dowel moment have been plotted for an infinitely long 0.75-in. dowel and a joint opening of 1 in. The pressures between the dowel and the concrete decrease very rapidly with each succeeding stress change.

The effect of introducing short dowels can be estimated as a correction factor. The procedure is indicated for 0.75-in. dowels, cut at the second point of pressure change. This point (see Table 1) is 5.81 in. to 5.36 in. from the joint face. According to Equations (6) and (7) the shear and the moment in an infinitely long dowel at this point, for a 0.75-in. dowel and 1-in. joint opening, are: Shear, − 17.5 lb; and moment, + 21.5 in.-lb.

These values are less than 5% of the values at the face of the joint, the moment being of reverse sign. The correction in the actual stresses, made necessary by cutting the dowel at this point, is equivalent to considering the dowel extending toward the left from the end loaded with a shear of + 17.5 lb and a moment of − 21.5 in.-lb. The pressure, shear, and moment due to these correction loads are shown by the broken curves in Fig. 3, as corrections to the first developed diagrams. As may be seen, the influence of cutting the dowel at the second stress-change point is not noticeable upon the maximum stress at the face of the joint, the computed influence being less than 0.25 per cent. It is unwarranted to repeat the correction process back from the face of the joint.

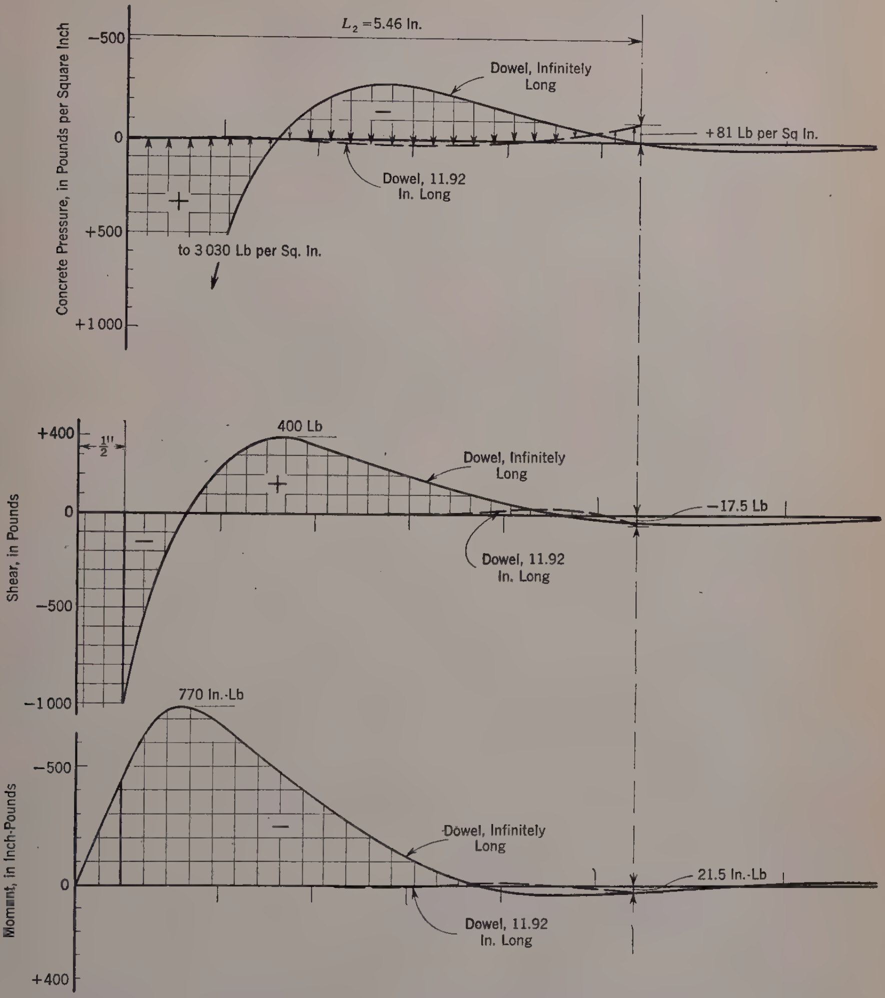

FIG. 3.—TYPICAL PRESSURE, MOMENT, AND SHEAR DIAGRAM FOR A DOWEL SHEAR OF 1 000 POUNDS

The computation indicates that it is entirely unnecessary to extend the dowels beyond the second point of pressure change. Since values of shear and moment are quite low for some distance before the second stress-change point is reached, the dowels could be made as much as 25% shorter without appreciably affecting the maximum stresses in and around the dowels.

DOWEL DEFLECTION ACROSS A JOINT

Of primary importance in judging the dowel action in concrete pavements is the deflection across the joint. The less this deflection, the more nearly will the adjacent pavement slabs match, and thus the greater part of the wheel load will be transferred by the dowel from the loaded side to the unloaded side.

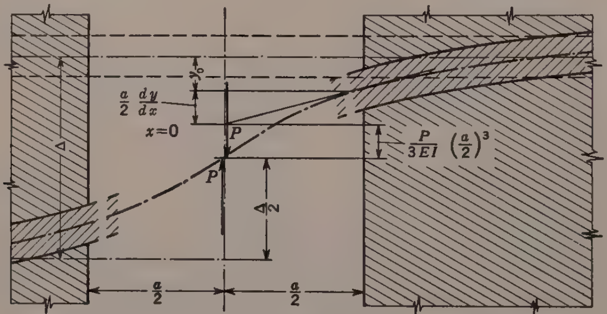

FIG. 4.—DEFLECTION OF A DOWEL ACROSS A PAVEMENT JOINT

For a given shear, the total deflection of a dowel is a function of: (1) The deflection at the faces of a joint; (2) the slope of the dowel, at the faces of the joint; and, (3) the deflection in the part of the dowel in the joint opening. The conditions are illustrated in Fig. 4, and may be defined by a simple formula under the assumption that the faces themselves remain parallel.

The deflection, Δ, across the joint is equal to twice the sum of the following quantities:

(*a*) The deflection at each joint face, y_0;

(*b*) The deflection due to the dowel slope over one-half the joint, $\frac{a\,dy_0}{2\,dx}$; and,

(*c*) The deflection of the dowel steel over one-half the joint, $\frac{P}{3\,E\,I}\left(\frac{a}{2}\right)^3$.

The deflection,. Δ, is obtained directly from Equations (9) and (10) by adding Increment (*c*); that is:

$$\Delta = \frac{P}{2\,E\,I}\left(\frac{1 + (1 + \beta\,a)^2}{\beta^3} + \frac{a^3}{6}\right) \ldots\ldots\ldots\ldots\ldots (14)$$

The deflection is proportional to the dowel shear, P, as shown for 0.75-in., 1-in., and 1.25-in. dowels in Table 2.

In practice, the joint faces would not remain parallel. The slope of the pavement slab in the direction of the dowel, as experimentally observed in tests of the United States Bureau of Public Roads,[6] however, rarely exceeds 10% of the slope of the dowel at the face of the joint as determined by Equation (10). For maximum dowel shear, when the wheel load is near the joint, the two joint faces may slope in opposite directions, minimizing the influence. Therefore, the effect of pavement slope upon dowel deflection across joints may be neglected safely.

TABLE 2.—DEFLECTION ACROSS JOINT FOR 1 000-POUND DOWEL SHEAR AND PARALLEL JOINT FACES

Diameter of dowel, in inches	DEFLECTIONS, IN INCHES, FOR THE FOLLOWING JOINT WIDTHS, a, IN INCHES					
	0	0.25	0.50	0.75	1.00	1.50
0.75	0.0043	0.0053	0.0065	0.0079	0.0094	0.0134
1.00	0.0026	0.0031	0.0036	0.0042	0.0049	0.0062
1.25	0.0018	0.0020	0.0023	0.0027	0.0030	0.0039

EFFECTS OF DOWEL SPACING ON STRESS RELIEF

The dowels, correctly designed, should serve the twofold purpose of decreasing deflections and of decreasing the stress in the concrete due to concentrated wheel loads. The maximum stress is that which occurs parallel to the edge—for edge-line loading a tensile stress in the bottom of the slab immediately under the load; and, for corner loading, a tensile stress in the top of the slab some distance from the corner. In each case, this stress is proportional to the moment in the concrete and, in turn, to the load supported by the concrete.

Influence of Dowel Spacing Upon Moments.—A dowel placed immediately beneath the load may be assumed to decrease the stress in the pavement in the same proportion as it transfers part of the wheel load to the adjacent slab by the dowel shear. Dowels some distance from the point of loading cannot be considered as effective as those that are closer. They will be farther from the point of maximum moment, and their action will be influenced by shear conditions in the concrete itself. The ideal dowel condition would be that which would transfer shear along the line of the joint identical with the interior of the slab. When support across the joint is lacking, the shear in planes parallel to the joint will increase correspondingly. The combination of shear in the concrete and dowel shear, accordingly, influences the moment and stresses in the concrete.

Convenient diagrams for determining the moment under and near the wheel load have been presented by H. M. Westergaard, M. Am. Soc. C. E.[7] For loadings a considerable distance from any edge the maximum positive moment is beneath the load, the maximum negative moment is 1.8 l from the point of load, l being the radius of relative stiffness as defined by Dean Westergaard. The moment diagrams for loading at an edge a considerable distance from any corner show a small maximum negative moment at approximately the same distance (1.8 l) from the point of loading.

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[6] "The Structural Design of Concrete Pavements," Pt. 4, *Public Roads*, September, 1936.

[7] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting, Highway Research Board, 1925.

At the points of maximum negative moment, the shear in the concrete in a plane perpendicular to the edge must equal zero. This shear is the sum of the forces on one side of the section: The subgrade reaction, the concrete shear in planes parallel to the joint, and the dowel shear. Beyond the point, 1.8 l, from the load, the moment changes very little. The influence of any one of the shear increments beyond that point, upon moment under the load, therefore, may safely be neglected. The effective dowel shear may reasonably be estimated in accordance with the following rule:

> The effective dowel shear decreases inversely as the distance of the dowel from the point of loading, to zero at a distance of 1.8 l. No dowels beyond that point influence the moment at the load point.

It is sufficiently accurate to consider the moment diagram of each dowel shear as identical to that produced by an equal wheel load, the dowel shear acting in an opposite direction to the wheel load. Table 3 gives the moment immediately under the wheel load of effective dowel shears for increasing distances from the point of loading to the dowel, the unit moments being taken directly from data presented by Dean Westergaard.[7]

TABLE 3.—PROPORTION OF EDGE MOMENT RELIEVED BY DOWEL SHEAR

Description	DISTANCE FROM THE LOAD POINT TO THE DOWEL AS A PROPORTION OF THE RADIUS OF RELATIVE STIFFNESS:							
	0	0.2l	0.4l	0.6l	0.8l	1.0l	1.4l	1.8l
Proportion of effective dowel shear	1.00	0.89	0.78	0.67	0.55	0.44	0.22	0
Moment at the Load Point, in Inch-Pounds:								
For a unit dowel shear	0.49	0.35	0.15	0.05	0.01	−0.02	−0.05	−0.06
For the effective dowel shear	0.49	0.31	0.12	0.03	0.005	−0.01	−0.01	0
Moment expressed as a proportion of the moment due to one dowel at the load point	1.00	0.64	0.24	0.07	0.01	−0.02	−0.02	0

From these data (Table 3), the relative effectiveness in decreasing slab stress may be developed numerically for any spacing, number, and arrangement of dowels relative to the load point. The sum of the effectiveness of the active dowels, expressed as percentages of the effectiveness of one dowel, immediately beneath the wheel load, for varying pavement depths, and for dowel spacings from 8 in. to 20 in., is illustrated in Fig. 5. The lowest efficiency occurs when the wheel load is between two dowels. With customary dowel spacing, wheel loads between dowels will occur most frequently (see solid lines in Fig. 5), and in rating the stress-relieving value of dowels relative to their spacing, they should be so compared. The average effectiveness for 5-in., 6-in., and 7-in. pavement thickness under these conditions is as follows:

Dowel spacing, in inches	Proportion of single dowel shear
8	1.55
12	1.00
16	0.60
20	0.35

If the dowels are to be effective in decreasing stresses in the concrete, they must be spaced close together.

These values, established directly from Table 3, are based on the assumption that the edge moment distribution is not changed. For very close dowel spacing and for very rigid dowels this assumption would be too approximate and the values must then be corrected for some moment distribution between that for edge loading and for interior loading. The values probably apply closely for dowel spacing of 12 in. with 0.75-in. and 1-in. dowels, and for a spacing of 12 in.; and also for 1.25-in. dowels at expansion joints.

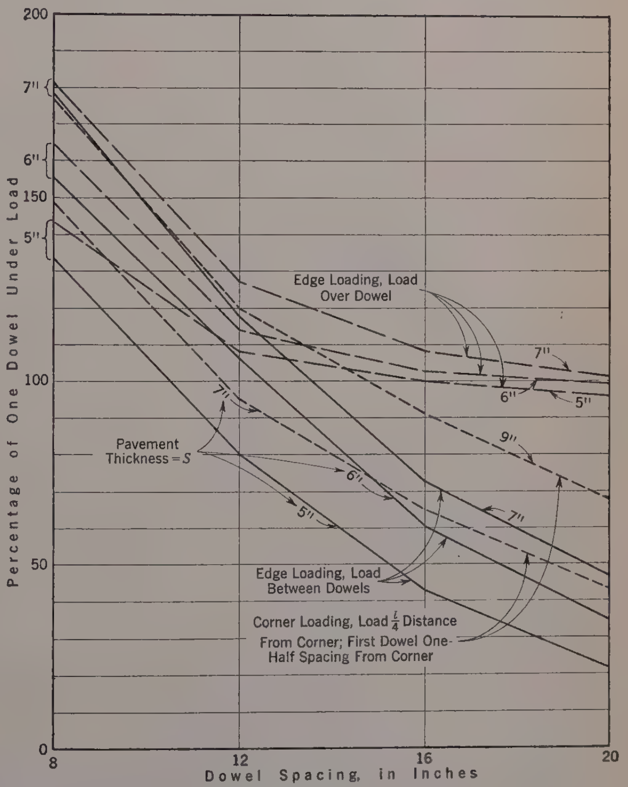

FIG. 5.—TOTAL EDGE STRESS RELIEF AS A PERCENTAGE OF THAT AFFORDED BY ONE DOWEL DIRECTLY UNDER A LOAD

Actual Stress Relief Due to Dowels Across Transverse Joints

Relation Between Pavement Stiffness and Dowel Stiffness.—For a complete evaluation of dowel effectiveness it is necessary to establish a basic value of one dowel at the loading point only. The rates of deflections of dowels have been established as shown in Table 2. The actual load transfer by a dowel directly under the load may then be determined if the deflection of the pavement edges or corners under a load is known.

The assumption that the deflection is directly proportional to the load carried by the concrete (see Fig. 6) approximates closely the results obtained in actual deflection measurements by the Bureau of Public Roads. Computed in accordance with methods established by Dean Westergaard the lines correspond to theoretical values of $k = 300$ for edge loading and $k = 200$ for corner loading.

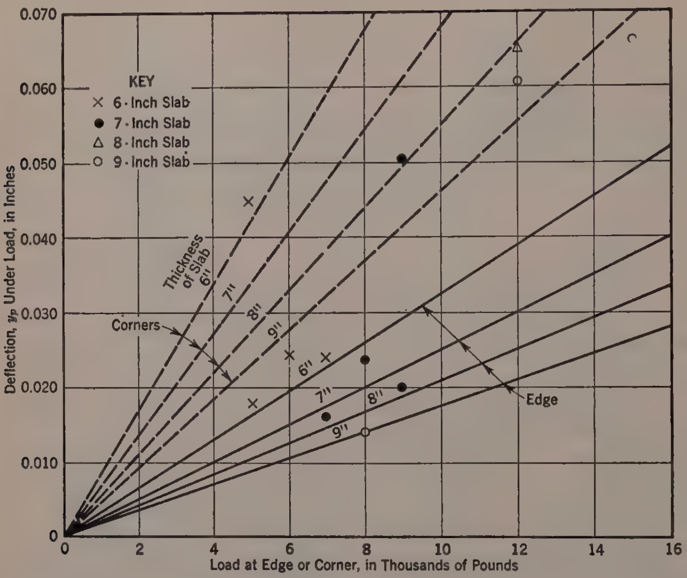

FIG. 6.—DEFLECTIONS OF PAVEMENT UNDER EDGE AND CORNER LOAD ON A SILT LOAM SUBGRADE

The deflection of the loaded pavement edge must equal the deflection of the dowel plus the deflection of the adjacent pavement edge, and the load on the adjacent pavement edge must be the same as the dowel shear. If W = the wheel load, P = the shear on a dowel directly under the load, and y_p and y_d = the deflection of the pavement and the dowel, respectively, for a unit load, the following values govern:

(*a*) The deflection of the loaded edge of a pavement, $y_p\,(W - P)$;
(*b*) The deflection of the dowel, $y_d\,P$; and
(*c*) The deflection of the adjacent edge, $y_p\,P$.

The sum of the deflections on either side must be identical, or:

$$y_p\,(W - P) = y_d\,P + y_p\,P \qquad (15)$$

from which equation the dowel load transfer is obtained, thus:

$$\frac{P}{W} = \frac{1}{2 + \dfrac{y_d}{y_p}} \qquad (16)$$

The values have been computed for common pavement dimensions in Table 4, with y_p obtained[8] from Fig. 6, and y_d from Table 2.

TABLE 4.—PERCENTAGE OF LOAD TRANSFER ACROSS TRANSVERSE JOINTS FOR PAVEMENTS OF STANDARD THICKNESS; (SILT LOAM SUBGRADE; ONE DOWEL DIRECTLY UNDER THE LOAD)

Width of joint, *a*, in inches	PERCENTAGES FOR THE FOLLOWING DOWEL DIAMETERS, IN INCHES:								
	(*a*) 6-Inch Slab; Joint Edge Deflection = 0.0033 × the Load in Kips*			(*b*) 7-Inch Slab; Joint Edge Deflection = 0.0025 × the Load in Kips*			(*c*) 9-Inch Slab; Joint Corner Deflection = 0.0045 × the Load in Kips*		
	0.75	1.00	1.25	0.75	1.00	1.25	0.75	1.00	1.25
0	30.2	35.8	39.5	26.8	32.9	36.8	33.8	38.8	41.8
0.25	27.7	34.2	38.2	24.2	31.0	35.6	31.5	37.3	40.8
0.50	25.3	32.4	37.1	21.8	29.1	34.3	29.2	35.7	39.8
0.75	22.8	30.5	35.6	19.5	27.1	32.6	26.7	34.0	38.6
1.00	20.6	28.7	34.2	17.3	25.2	31.1	24.4	32.3	37.4
1.50	16.5	25.8	31.4	13.6	22.3	28.0	20.1	29.6	34.9

* 1 kip = 1 000 lb.

Table 4, together with Fig. 5 (actual dowel spacing effect computed for 8-in. to 20-in. spacing), affords an opportunity of checking the theoretical value of stress relief due to dowels with experimental data. The percentage of load transferred, as shown in Table 4, is obtained only if the wheel load is directly over the dowel and if the stresses in the dowel and the surrounding concrete do not exceed safe values.

TABLE 5.—COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL VALUES OF DOWEL STRESS RELIEF (0.75-INCH DOWELS)

Position of load	6-INCH PAVEMENT; 0.75-INCH JOINT; 0.75-INCH DOWELS; 18 INCHES, CENTER TO CENTER						7-INCH PAVEMENT; BUTT-JOINT; 0.75-INCH DOWELS; 24 INCHES, CENTER TO CENTER					
	Observed Stress Relief				Computed Stress Relief		Observed Stress Relief				Computed Stress Relief	
					One Dowel Under Load 22.8% (Table 4)						One Dowel Under Load 16.8% (Table 4)	
	Stress Under Load, in Pounds per Square Inch, with Edge		Stress relief, in pounds per square inch	Stress relief (percentages)	Effectiveness for load position and spacing, Fig. 5 (percentages)	Adjusted stress relief computed (percentages)	Stress Under Load, in Pounds per Square Inch, with Edge		Stress relief, in pounds per square inch	Stress relief (percentages)	Effectiveness for load position and spacing, Fig. 5 (percentages)	Adjusted stress relief computed (percentages)
	Not doweled	Doweled					Not doweled	Doweled				
Between dowels...	297	260	37	12.5	48	11	275	215	60	22	30	8
At dowel.........	297	235	62	21.0	101	23	275	182	93	34	100	27

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[8] *Public Roads*, October, 1936, p. 176 *et seq.*

Accurate measurements have been reported [8] by L. W. Teller, Assoc. M. Am. Soc. C. E., and Mr. E. C. Sutherland, from which comparative values have been computed in Table 5. The values in Table 5 for a 0.75-in. open joint check closely. For the 7-in. slab with butt-joint, the theoretical values are considerably lower than those developed experimentally, particularly for a wheel load between dowels. These experimental data indicate that there is considerable load transfer and joint restraint due to surface friction across the butt-joint, which will make the experimental values of stress relief greatly higher than what would be obtained with dowels alone.

For open contraction joints and expansion joints of conventional types, the foregoing theoretical method of computing stress relief due to load transfer apparently gives results, in substantial agreement with those observed experimentally. This is evidence that the assumptions in regard to concrete yield, K, and the influence of dowel spacing by the use of edge moment distribution, are fairly correct.

The method offers a possibility of determining the performance of dowels in regard to size and spacing, and to furnishing designs for any required degree of stress relief across a doweled edge. In no applicable case can the load transfer or stress relief exceed 50 per cent. For the close spacing of rigid dowels the method requires adjustment, because in that case it is not a sufficiently close approximation to assume that the moment distribution for an unsupported edge governs for a doweled edge.

STRESSES IN AND AROUND DOWELS

Table 1 gives the concrete bearing stresses around a 0.75-in. dowel for a shear of 1 000 lb. The moment in the dowel for bearing stresses from 2 160 lb per sq in. to 3 460 lb per sq in. varies from 400 in.-lb to 980 in.-lb, and the maximum fiber stress in the dowel varies from 9 700 per sq in. to 23 600 per sq in. Neither of these stresses exceeds that which can easily be taken by the materials; of the two, probably the concrete bearing stress is the critical stress. In joints from 0 in. to 0.75 in. wide and for normal subgrade conditions, 0.75-in. dowels may be expected to transfer not more than about 25% of the wheel load in common pavements as shown in Table 4. A dowel shear of 1 000 lb would then correspond to a single tire load of 4 000 lb. These values are in fair accord with those encountered in actual practice. If the dowels connect pavement slabs under which subgrade support is inadequate, they will be required to transfer more nearly one-half the wheel load, in which case stresses approximately double those for a 1 000-lb dowel shear may be expected to occur. The extreme stresses would then approach both the yield point in the steel and the ultimate compressive strength of the concrete as established by the usual tests, although, in the confined space surrounding the dowel, the ultimate concrete stresses may be considerably higher than those established in the usual tests.

A change from 0.75-in. to 1-in. dowels would improve the load transfer across joints from a 25% to a 33% maximum in accordance with Table 4. The concrete stress beneath the dowel for a 1 000-lb dowel shear would vary from 1 300 to 1 950 lb per sq in., the maximum moment in the dowel from about

495 in.-lb to 1 060 in.-lb, and the extreme fiber stress in the steel from 5 000 to 10 800 lb per sq in. The increased load-transfer capacity would noticeably strengthen the joint edges both against corner cracking and longitudinal cracking between the edge and the center joint. The decrease in stress both in the concrete and in the steel, with the use of 1-in. dowels, is such that their use in heavy-duty concrete pavements seems well warranted.

CONSTRUCTION CONDITIONS AND DOWEL PERFORMANCE

Effect of Inaccurate Dowel Alignment.—If all dowels across a transverse joint are parallel and the concrete slabs remain parallel, there is no resistance to movement in the joint except that exerted by frictional bond between the dowel steel and the concrete. If the dowels are not in alignment, however, their resistance to the movements of the joint will increase in proportion to the severity of the movements. Forces will arise to deflect both the dowels and the pavement edges sufficiently to make their deflection correspond to the relative movement. These are shear forces on the dowels, and their deflection may be computed according to established formulas, as illustrated in Fig. 4. Each one of the two deflections—that of the dowels and that of the pavement—is ascertainable independently. With common dowel spacings, the stiffness of the pavement edge will be so much greater than the stiffness of the dowels that the deflection of the pavement between adjacent dowels may be neglected.

The dowel deflection is given in Equation (14). In its application to a dowel on an angle with the direction of movement, the dowel deflection must equal the transverse component of the movement in a parallel displacement of the dowel; this is proportional to the movement and to the angular deviation of the dowel.

Under the assumption of a fixed pavement edge, the relationship between the deflection of the joint and the dowel misalignment then is:

$$\Delta = \frac{P}{2\,E\,I}\left(\frac{1 + (1 + \beta\,a_i)^2}{\beta^3} + \frac{a_i^3}{6}\right) = \alpha\,i \quad \ldots\ldots\ldots\ldots (17)$$

in which i = additional opening of the joint; a_i = new joint width; and α = misalignment of the dowel in the direction of movement, in radians. The relationship between the developed dowel shear, P_s, due to the misalignment, α, therefore, may be written:

$$P_s = \frac{2\,E\,I\,\alpha\,i}{\dfrac{1 + (1 + \beta\,a_i)^2}{\beta^3} + \dfrac{a_i^3}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Table 6 contains computed shear values for various joint widths, and joint movements. The data apply for dowel misalignment in the horizontal as well as in the vertical direction, provided the modulus of support in the concrete, K, is the same in both directions.

With the aid of Table 6, it is possible to find the loads on the dowels for any given construction condition. Shears and bearing stresses are shown for contraction joints and expansion joints in Fig. 7. If the misalignment should be greater than 1% (approximately $\frac{1}{8}$ in. per ft of dowel), the dowel deflection necessary for readjustment will be proportionately greater. The resulting dowel loads and stresses will increase proportionately.

TABLE 6.—SHEAR, P_s, IN POUNDS, DUE TO A DOWEL MISALIGNMENT OF 1% ($\frac{1}{8}$ INCH PER FOOT OF DOWEL)

Final joint width, a_i, in inches	CHANGE IN JOINT WIDTH, i, IN INCHES					
	0.25	0.50	0.75	1.00	1.50	3.00
	Corresponding Vertical Dowel Deflection, in Thousandths of an Inch					
	2.5	5.0	7.5	10.0	15.0	30.0
(*a*) 0.75-INCH DOWELS						
0	580	1 155	1 730	2 310	3 460	6 920
0.25	470	940	1 410	1 880	2 820	5 640
0.50	390	775	1 160	1 550	2 320	4 640
0.75	320	640	955	1 275	1 910	3 820
1.00	265	530	795	1 060	1 590	3 180
1.50	185	370	560	745	1 120	2 240
3.00	75	150	230	305	460	920
(*b*) 1.00-INCH DOWELS						
0	960	1 920	2 880	3 840	5 760	11 520
0.25	815	1 630	2 445	3 260	4 890	9 780
0.50	695	1 390	2 085	2 780	4 170	8 340
0.75	595	1 185	1 780	2 370	3 560	7 120
1.00	510	1 015	1 525	2 030	3 050	6 100
1.50	405	805	1 210	1 610	2 420	3 220
3.00	175	350	525	695	1 050	2 100
(*c*) 1.25-INCH DOWELS						
0	1 415	2 825	4 240	5 650	8 480	16 960
0.25	1 235	2 465	3 700	4 930	7 400	14 800
0.50	1 090	2 175	3 165	4 350	6 330	12 660
0.75	940	1 875	2 815	3 750	5 630	11 260
1.00	825	1 645	2 470	3 290	4 940	9 890
1.50	640	1 275	1 915	2 550	3 830	7 660
3.00	325	645	970	1 290	1 940	3 880
....	..	....	....	....	....	....

Since the bearing stress between the dowel and the concrete apparently is the critical stress, it has been developed from Table 1 and plotted in Fig. 7. As may be seen, exceedingly high concrete stresses accompany excessive dowel misalignment in transverse joints. Dowel loads from this source may exceed those caused by the normal load transfer function.

The actual forces will be less than those developed by assuming that the pavement is entirely rigid. To some extent, the direction of longitudinal movement follows the average alignment of all dowels. The force acting in the dowel, therefore, is dependent on the individual deviation from this average. The full dowel force is not developed if the deviation in alignment occurs gradually from dowel to dowel.

Strength of the Concrete Around the Dowel.—Some general observations of concrete failures around 0.75-in. dowels have been reported by Mr. A. R. Smith and S. W. Benham, Assoc. M. Am. Soc. C. E.[9] The data are classified for study in Table 7, in which the stresses to which the misalignments correspond, as read from Fig. 7, are given in Column (5).

The uniformity of the two sets (in Table 7, Items Nos. 1 and 4 against Items Nos. 2, 3, and 5) of computed stresses for the observed evidence points

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[9] "Effect of Dowel-Bar Misalignment Across Concrete Pavement Joints," by Arthur R. Smith and Sanford W. Benham, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1133.

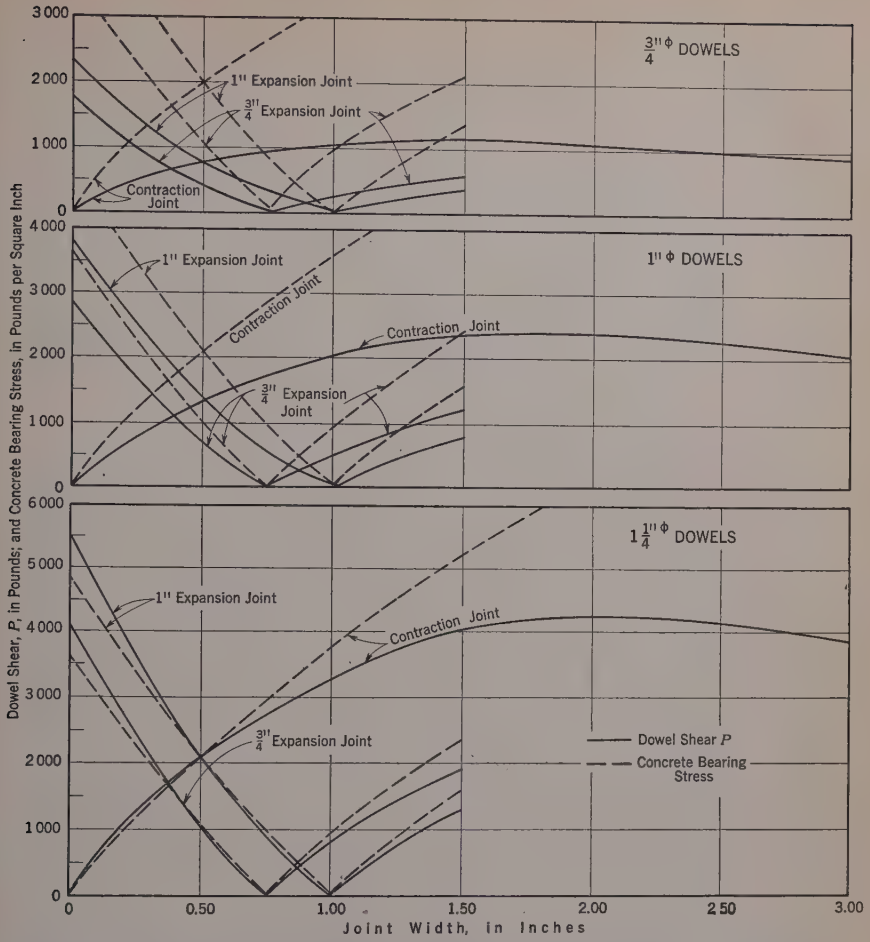

Fig. 7.—Dowel Shear and Concrete Stress Produced by Joint Movement Deflecting a Dowel 1 Per Cent. (⅛ Inch per Foot of Dowel)

TABLE 7.—Observed Concrete Strength Around Dowels

Item No.	Type of joint (1)	Degree of misalignment (2)	Additional opening,* *i*, in inches: Initial (3)	Final (4)	Extent of spalling around dowels (5)	Concrete bearing stress, in pounds per square inch, from Fig. 7 (6)
1	Contraction	1%	0	0.75	Slight	2 700
2	Contraction	4%	0	0.75	Distinct	10 800‡
3	Contraction	6%	0	0.50	Failure	12 000
4	Expansion	1%	1.00	0.25*	Slight	3 300
5	Expansion	4%	1.00	0.25*	Complete†	13 000

* Additional closing at expansion joint. † Complete cracking and spalling. ‡ 4 × 2 700 = 10 800.

to distinctly separate stress conditions in the concrete. The slight spalling, with bearing stresses of about 3 000 lb per sq in., probably indicates failure of a secondary frictional nature, or direct tension immediately beside the dowel. The ultimate stress of more than 10 000 lb per sq in., undoubtedly indicates failure in compression, or for thin pavements failure in tension along the cone-shaped surface under or over the dowel. The behavior, including ten extensive cycles of movement, indicates that it may be safe, in design, to use a bearing stress of somewhat less than 3 000 lb per sq in. between the dowels and the concrete. The data indicate the presence of a high compression stress in the confined concrete surrounding the dowels, which had been expected. Higher stresses under exceptional conditions may be permissible because, although concrete failure would result around an exceptionally faulty dowel, such failure may not be noticeable in a pavement failure on good subgrade.

PERMISSIBLE LIMITS OF DOWEL MISALIGNMENT

Theoretical analysis of stresses due to dowel misalignment establishes the fact that the dowels must be placed with considerably greater accuracy for wide joints than for narrow joints if both are to close to the same value of a_i. Fig. 7 also illustrates the fact that 1-in. dowels cause no greater bearing stresses in the concrete than 0.75-in. dowels for identical misalignment. If the limiting stress due to the average misalignment commonly encountered in practice is set at 2 000 lb per sq in. in the concrete, the misalignment of 0.75-in. expansion joints intended to maintain a minimum opening of 0.25 in. should not be more than 1% for either 0.75-in. or 1-in. dowels. The same permissible average dowel misalignment would apply to 0.75-in. or 1-in. dowels extending through a contraction joint with a maximum opening of 0.50 in.

It would seem impossible to tolerate alignment errors of 0.5 in. per ft of length, except under the most exceptional circumstances. The efficiency of the joint and of the dowels is seriously impaired by greater misalignment than $\frac{1}{8}$ in. per ft, although the edges of the joint may not fail.

PRACTICAL PROBLEMS IN ALIGNMENT

Most pavement designs include spacer or distribution bars parallel to, and on each side of, transverse joints, to which bars the dowels are tied. The spacer bars are justified for reinforcement at the edge of the joint. However, wiring has not proved adequate to maintain the horizontal alignment of the dowels during the construction operations. Some pavement designs, accordingly, specify crimping or other mechanical processes, to locate the dowels exactly and also to extend the spacer bars exactly to the side forms so the dowels in both pavement lanes are parallel with the longitudinal axis of the pavement. The horizontal alignment can be made exact in that manner without increasing the material or work necessary in the construction of the pavement.

Satisfactory alignment in the vertical direction can be obtained with supports from the subgrade placed near the outer ends of each dowel. These supports must be substantial enough to retain their position during concreting. It should be possible to adjust the dowels with a spirit-level if inspection reveals them to be out of line more than the permissible tolerance. A great number of observations in the field have revealed that, with reasonable care and inspection, dowels supported near their ends may easily be placed with misalignments of less than $\frac{1}{8}$ in. per ft.

Dowels supported on the joints only, or very near the joints, cannot easily be maintained in satisfactory alignment, because they will be influenced by movements of the joint under the action of the fresh concrete. Particular care is required to maintain short dowels in correct alignment as the vertical deviation at their ends is not at once discernible in casual inspection.

CONCLUSION

Dowels across the transverse joints of pavements can be designed exactly by simple formulas. The value of K (1 000 000 lb per in.^{3}) used in this paper should be verified experimentally by actual load tests on dowels. It is believed to be sufficiently close, however, to permit its use for stress determination. Actual stress determinations show relatively high stresses both in and around 0.75-in. dowels under loads that may be encountered in practice. Increase in the size of dowels relieves the concrete stress quickly. The dowel length may be decreased to one-half the 24-in. length now common in actual construction.

Experimental and theoretical stress and moment determinations in pavements subjected to edge loading indicate that dowels which are farther away than 1.8 times the radius of relative stiffness are totally ineffective for the relief of traffic load stresses and that their effect decreases rapidly with increasing distance from the point of load, within the effective distance. Linear decrease in effective shear is proposed as a convenient and sufficient approximation.

Dowel deflection across the joints is primarily a function of dowel deflection and elastic slope at the face of the joint. Deflection of 1-in. dowels is approximately one-half the deflection of 0.75-in. dowels. The stress relief offered by single dowels is dependent on their deflection, and the rate of deflection of the pavement on the subgrade. By proper design and construction, stresses can be reduced 25% to 35% at the average joint opening. Dowels are effective for this purpose if they are spaced sufficiently close. The performance of a dowel can be predicted quickly by the method given in this paper, and the size of the dowel, as well as the proper spacing, can be determined for any desired stress relief. The results check closely the values that have been determined experimentally.

Stresses due to inaccurate dowel installation have been determined. With extreme joint movements, they may exceed the stresses due to traffic loads unless care is given to this construction detail. Observed concrete failures under known conditions of dowel misalignment indicate the presence of high ultimate concrete compression stresses in the confined space around the dowels. A design stress of 2 000 lb per sq in. between the dowel and the concrete appears justifiable. Dowels should be aligned with extreme care if they are to perform their function during the life of the pavement. Satisfactory dowel performance can be obtained with less weight of steel than is now used, but as the dowels are shortened, increasing care must be given the exactness of the supports and spacing of the dowels.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[10] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

a = width of a joint between pavement slabs; a_i = new width after the joint has spread or closed;
b = breadth = the width of a structure embedded in an elastic mass;
E_s = modulus of elasticity of a structure; E_c = modulus of elasticity for concrete;
e = base of Naperian logarithms;
I = moment of inertia of a structure;
i = additional opening or closing of a joint from its original dimension;
K = modulus of support in an elastic mass, defined as the pressure intensity, in pounds per square inch, on a mass, sufficient to cause a settlement of 1 in.;
k = modulus of subgrade reaction;
L = length; $L_1, L_2, \cdots L_n$ = distances to Sections 1, 2, $\cdots$ n, from the face of an elastic mass, Sections 1, 2, $\cdots$ n being points where the pressure changes direction;
l = radius of relative stiffness of the pavement;
M = moment at any point of a structure and an elastic mass; M_0 = moment in a structure at the left face of an elastic mass;
P = load, or shear, on a structure at the face of the elastic mass; P_s = the load equivalent to the developed shear;
s = unit stress; s_ρ = unit stress at radial distance, ρ;
V = shear in the structure at any point;
W = total weight; wheel load;
x = distance parallel to the X-axis, or the horizontal axis of a structure; x_m = point in a dowel where maximum movement occurs;
y = deflection; distances parallel to the Y-axis; deflection of a structure relative to an elastic mass; y_0 = deflection at the face of an elastic mass; y_p = deflection of a pavement acted upon by a unit load; y_d = deflection of a dowel acted upon by a unit load;
α = misalignment of a dowel, in radians;
β = relative stiffness of a structure embedded in an elastic mass;
Δ = total deflection across a joint;
θ = angular distance corresponding to radius of stiffness, ρ;
ρ = radius of stiffness.

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[10] A.S.A.—*Z*10*A*—1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES

By T. H. Rust,[1] Assoc. M. Am. Soc. C. E.

Synopsis

An attempt is made in this paper to supply information about the structural steel plates designed to transfer force from one member to another in a structural steel frame or bridge. The forces enter and emerge from these gusset-plates, often by means of rivets, but sometimes through welds. Since a gusset-plate is a point of discontinuity of stress transfer in a structural steel frame, it is here that failures most often occur.

As early as 1867, J. W. Schwedler noted [2] the unequal distribution of stress among the rivets in a riveted connection. He found that the end rivets took most of the stress and the interior ones least. Likewise, the ends of a welded connection are more highly stressed than the interior.

This inequality of stress transfer into a gusset-plate can be shown by stressing models of the type shown in Fig. 1, which are carved from single pieces of bakelite to simulate gusset-plates. When forces are applied through the legs to the plate, fringed photographs can be made by the aid of photo-elasticity. The photo-elastic fringes are the means of establishing the stress value $(p - q)$, at a particular point, p being one principal stress and q the other principal stress at right angles to it. Although a $(p - q)$-determination is a good indication of the condition of stress in a plate, a $(p + q)$-determination was made for some of the models. The lateral expansion or contraction of the stressed plate at any point is proportional to the $(p + q)$-stress at that point. This change in thickness was measured by a light interferometer. From the $(p - q)$ and the $(p + q)$-stress determinations, the value of the principal stresses, p and q, can be found.

Studies of several different shapes are reported herein and the stresses encountered are discussed. The models were designed with special reference to

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NOTE.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **January, 1939**, *Proceedings*.

[1] Chicago, Ill.

[2] "Ueber Vernietungen," von J. W. Schwedler, *Deutsche Bauzeitung*, Architekten-Vereins zu Berlin I, November 29, 1867, No. 48, S 463–465.

gusset-plates. A part of the paper is devoted to recommendations for the design and specification of steel gusset-plates.

TESTS

Tensile forces were applied to the model shown in Fig. 1 by means of steel pins through the bushings. A force enters through the top pin, passes down the upper legs into the plate and is resisted by a reaction passing through the lower legs to the other pin, which is held in a stationary position. As shown in the side view of Fig. 1, the bushings are beveled to keep the force, *P*, on the center line of the model. At all junctions of the legs and the plate small fillets were provided to minimize local stress concentration. The plates were each about 0.2 in. thick, and the legs were attached to the plate for a distance of 0.75 in.

To avoid residual stresses in the edges of the bakelite plate, the models were annealed and polished as rapidly as possible. They were then subjected to forces, *P*, in the path of polarized light (see Fig. 2). A theory is that polarized light has its vibrations restricted to one plane, in contrast with ordinary light, in which light vibrations are said to be in many planes. When a beam of ordinary light is passed through a Nicol prism the light vibrations are held to a single plane, and it is then known as polarized light. If this light beam is passed through a second Nicol prism with its axis at right angles to the first prism, the beam disappears, and the field becomes dark. Since there are no vibrations perpendicular to one plane in a beam of interrupted polarized light, the second prism (termed the "analyzer") cannot transmit any light because its axis is at right angles to the axis of the first Nicol prism or polarizer.

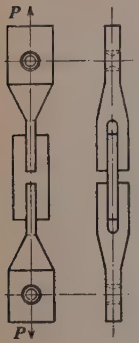


FIG. 1

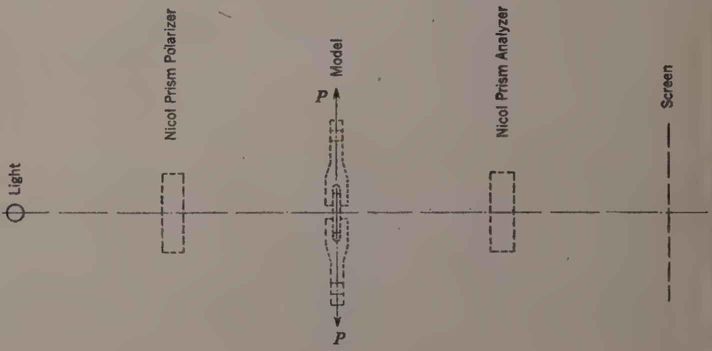


FIG. 2

If a polished bakelite model under stress is placed between the two prisms in the path of the polarized light, the second Nicol prism, or analyzer, transmits light between dark fringes. All the points on a particular dark fringe are of equal $(p - q)$-stress in the plate of a model. Monochromatic mercury light was used.

The change in $(p - q)$-stress from one fringe to another is constant for a given thickness of plate. This stress change between fringes is determined by subjecting a pilot specimen of the bakelite to known loads.

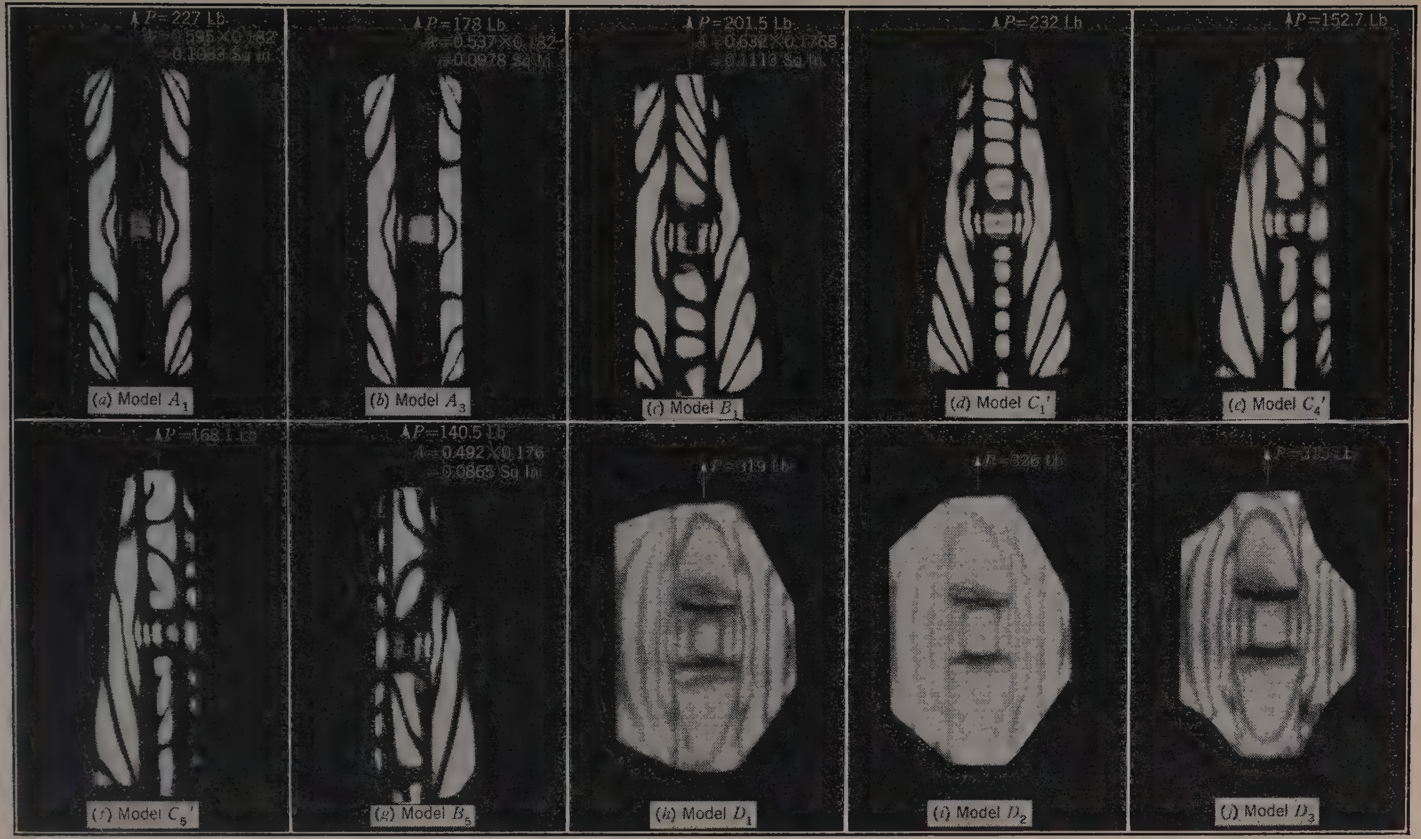


FIG. 3.—FRINGED PHOTOGRAPHS OF PLATES OF DIFFERENT SHAPES

Fig. 3 shows fringed photographs for plates of different shapes. The plates of Models A_1, B_1, and C_1' were $\frac{5}{8}$ in. wide at the mid-length; for Models D_1, D_2, and D_3, the plates were 1.25 in. wide at the mid-length. For Models A_1, B_1, C_1', D_1, D_2, and D_3, the area of the plate at mid-length was nearly equal to the area of the two legs. The closeness of the fringes near the ends of the plates, at the legs, shows the rapid transfer of stress into the plates at those points. The unit stress in the legs is large at this point, whereas in the ends of the plate at the legs the unit stress is small. The difference in the unit stress of the legs and the plate causes rapid stress transfer. At the mid-length of the legs, the bakelite in the legs is stressed to about the same value as the plate. The comparatively great distance between the fringes in the plate at the mid-length of the legs, shows the less rapid transfer of stress at this point, Near the ends of the legs the unit stress is small in the legs; whereas the unit stress and unit deformation in the plates are large. This change in deformation accounts again for a rapid transfer of stress into the plate. The shear stress between the plate and the legs can be represented by a curve shown in Fig. 4. A smooth curve as indicated by the full line would be shown by a welded connection or a continuous connection such as exists in the models. A line of rivets would indicate a stepped curve as shown by the dotted lines in Fig. 4. A rivet or a row of rivets perpendicular to the length of the member would transfer an increment of shear into the plate equal to the area under the corresponding step.

If the cross-sectional area of the legs is equal to the area of the plate, the shear stress transfer at the end of the plate and at the ends of the legs would be of the same value. On the other hand if the area of the legs is less than the area of the plate there will be a greater transfer of stress into the plate at the end of the plate because of its greater stiffness, with correspondingly less transfer of stress at the ends of the legs, as shown in Fig. 5. If the legs were larger in area than the plate, the end ordinates of the curve would be reversed, as shown by the dotted curve.

Boundary Stresses.—At the outside boundary of a plate, symmetrical about the legs (as shown by Models A_1, B_1, and C_1', Fig. 3), the change of stress is less rapid than it is next to the legs. If this were not so, the $(p - q)$-fringes would extend from the sides of the legs, straight across the plate, to the boundary. The extension of the fringes, forward toward the mid-length of the plate, shows how stress lags at the boundaries of the plate. Corners of the plate are points of least accessibility of stress and the positions of zero stress. These corners, therefore, are good places to begin counting the fringes; each succeeding fringe represents a constant increase in $(p - q)$-stress over the preceding fringe.

The boundaries at the mid-length of the plate are the most highly stressed. The stress at that point also lags considerably behind the stress in the center of the plate at the corresponding positions. At the mid-length of the legs in Model A_1, Fig. 3 (which is approximately at the quarter-point of the plate), the fringes are nearly perpendicular to the boundary. There is comparatively little transfer of stress into the plate from the legs at this point, with less disposition of the stress to lag at the boundary. The same boundary stress condition is shown in Model C_1', Fig. 3, at the third fringe from the top, and again less definitely at the third fringe from the bottom. The wide fringe on the wide side of Model C_4' at the position of the upper leg shows that the increase in area of the plate practically compensates for the transfer of stress from the legs, causing little change in $(p - q)$-stress in the plate at this point on the boundary. The third fringe from the top on the narrow side of the plate turns back. This is caused by the bending moment due to the eccentricity of the plate. The bending stress more than compensates for the tendency of the stress to lag at the boundary. The fourth fringe of Model C_4' turns forward again because the more rapid transfer of stress from the legs near the ends causes the boundary stress to lag again. The same phenomena are shown in Models C_5', A_3, and B_5.

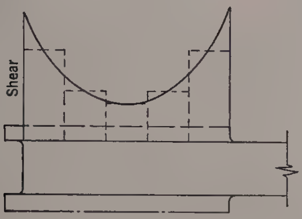


FIG. 4

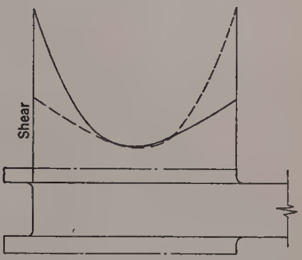


FIG. 5

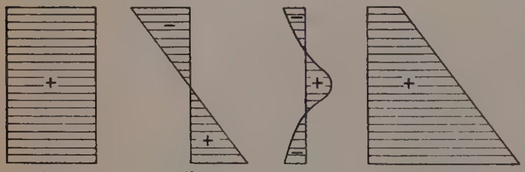
In order to show something about the plate corner effects, the corners on the left side of Model D_1, Fig. 3, were filed off for one-fourth the distance of the leg for Model D_2. There is a more rapid change of boundary $(p - q)$-stress near the left side of the leg in the plate of Model D_1 than in Model D_2. Due to greater resistance to stress transfer in Model D_1 than in Model D_2, the stress builds up more rapidly near the leg in Model D_1 than in Model D_2. The upper left-hand corner of Model D_1 has a larger area free of stress than Model D_2. Model D_1 has less continuity of stress transfer than Model D_2 in the upper left boundary. The $(p - q)$-stress in the right side of the gusset-plate shows almost no change from the removal of the corners on the left side. The upper outside corner of the right side of the plate is at the end of the leg, whereas the lower outside corner is only three-fourths the way along the leg toward its end. The $(p - q)$-fringes do not show any appreciable lack of symmetry due to this difference. The second fringe from the right on the right side of the plate bends out sharply near the boundary. This is true of the second fringe of all the *D*-models; for Model D_3, Fig. 3, the third fringe also bends out sharply at the boundary. The rapidly increasing area of the plate is more than sufficient to counteract the addition of stress from the legs at these points. It should be noted that the stress transfer from the legs is a minimum near the mid-length of the legs.

Model D_3, Fig. 3, was made by filing concave cuts in Model D_2. In these models there is less unit shear on the right side of the legs near the end of the plate than on the left side of the legs, because there is less material there and, therefore, less resistance. The smaller unit shear causes less shear stress in the end rivets or ends of welds, as well as less bearing stress on the plate from the end rivets. Since the rivets at the end of a gusset-plate as well as their bearing stresses on the gusset-plate are critical, it is well to make the plate comparatively elastic by not having its surface area much larger than the end rivets require.

The vertical boundaries of the plate of the *D*-models are almost free of stress. These boundaries are so far from the direct path of stress transfer that they are not highly stressed, in accordance with the theory of least energy.

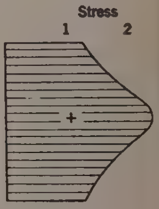
Interior Plate Stresses.—The greatest stresses in the interior of the plate are always between the ends of the legs of the models. All the stress has been transferred into the plate in this region, and it tends to concentrate in the most direct path of transfer. Some of the fringes curve out, which shows the tendency of the $(p - q)$-stress to spread over a larger part of the plate.

The stress condition in a plate may be shown graphically by the diagrams in Fig. 6, that is, Fig. 6(*a*) represents the direct stress on a transverse section assuming uniform distribution; Fig. 6(*b*) represents the bending stress; and, Fig. 6(*c*) represents the stress that minimizes the energy in the plate. The theory of least energy is not satisfied by the assumption of uniform and direct bending stress distribution, but requires that the assumption shall be modified by stress similar to that shown in Fig. 6(*c*). Although there is ample evidence in these tests shown graphically by Fig. 7, plotted from readings on the section at the mid-length of the plate of Model A_1 (Fig. 3), there is no generalized theory for the least energy stress condition for Fig. 6(*c*). If the outside forces are applied somewhere near the neutral axis of the plate this added least-energy stress will always have the general shape of the curve in Fig. 6(*c*), the negative area being equal to the positive area. Figs. 6(*d*) and 6(*e*) will have the area of Fig. 6(*a*), because equilibrium of the free body is always maintained between external and internal forces.



(*a*) (*b*) (*c*) (*d*) (*e*)

FIG. 6



(*f*)

FIG. 7

In nearly all the models, the fringes that pass near the ends of the legs change curvature. Since the largest concentration of $(p - q)$-stress occurs in the plate between the legs, this stress tends to distribute itself over a greater part of the plate with the resulting change of curvature of these fringes.

With an eccentric plate such as that shown in Model C_5' the fringes denoting the highest $(p - q)$-stress shift over toward the narrower side of the plate. The direct stress, the bending stress due to eccentric loading, and the stress due to the least energy in the plate, create a maximum stress to the right of the center line of the legs. In Model D_3, Fig. 3, there is also a slight shifting of the fringes of highest value to the right of the center line of the legs. On the right side of this model there is less plate in the gusset corners than on the left side of the legs; therefore, the maximum stress shifts again slightly to the right.

In Model C_5' the almost horizontal fringes on the narrow side of the plate show that the bending stress due to eccentric loading is approximately equal to the decrease in least energy stress there. The principal stress, q, is not large except in the mid-length of the plate; even there it is considerably less than the p stress, as will be shown under the heading, "Discussion of Curves."

DISCUSSION OF CURVES

In addition to knowing something of the $(p - q)$-stress distribution from the photo-elastic photographs of the models, it was desirable to know something also about stress distribution in gusset-plates from the plotting of stress curves.

Graphs of $(p - q)$-stress and $(p + q)$-stress along the sides of the legs of Model A_1 are shown in Fig. 8. The $(p - q)$-stress difference between fringes was found by experiment to be $\frac{84.6}{0.183} = 470$ lb per sq in.; 84.6 lb per sq in. would be the stress difference between fringes in case the bakelite were 1 in. thick. This constant divided by the thickness of the plate used in Model A_1 gives the value of $(p - q)$-stress difference between fringes. The positions of the $(p - q)$-fringes together with their successive values were plotted for Model A_1 in Fig. 8.

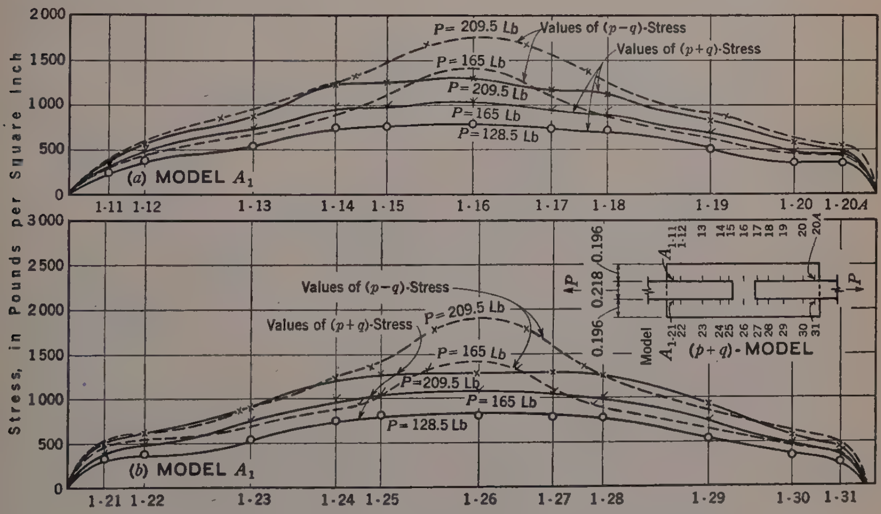


FIG. 8.—$(p - q)$ AND $(p + q)$ STRESS CURVES, MODEL A_1

The $(p + q)$-curves were plotted from values obtained by measuring the lateral contraction of the stressed plates at various points along the legs. The lateral contraction or expansion at a point of the stressed plates is proportional to the $(p + q)$-stress at that point. This lateral change in thickness of the bakelite plate was measured by a light interferometer,[3] and the resulting stresses were plotted as shown. The $(p + q)$-curves and the $(p - q)$-curves nearly agree except at the mid-length of the plate, which indicates that there is little q-stress except at the mid-length of the plate. Since there is a large concentration of stress directly between the ends of the legs, an appreciable q-stress arises in the effort to distribute the stress over a larger part of the plate. The difference in ordinates between the $(p + q)$-stress and the $(p - q)$-stress curves for a particular load, divided by 2, gives the q-stress for a particular point. The addition of the ordinates, divided by 2, gives the p-stress at a particular point.

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[3] "An Application of the Interferometer Strain Gage to Photoelasticity," by R. W. Vose, Jun. Am. Soc. C. E., *Journal of Applied Mechanics*, September, 1935, pp. A 99–102.

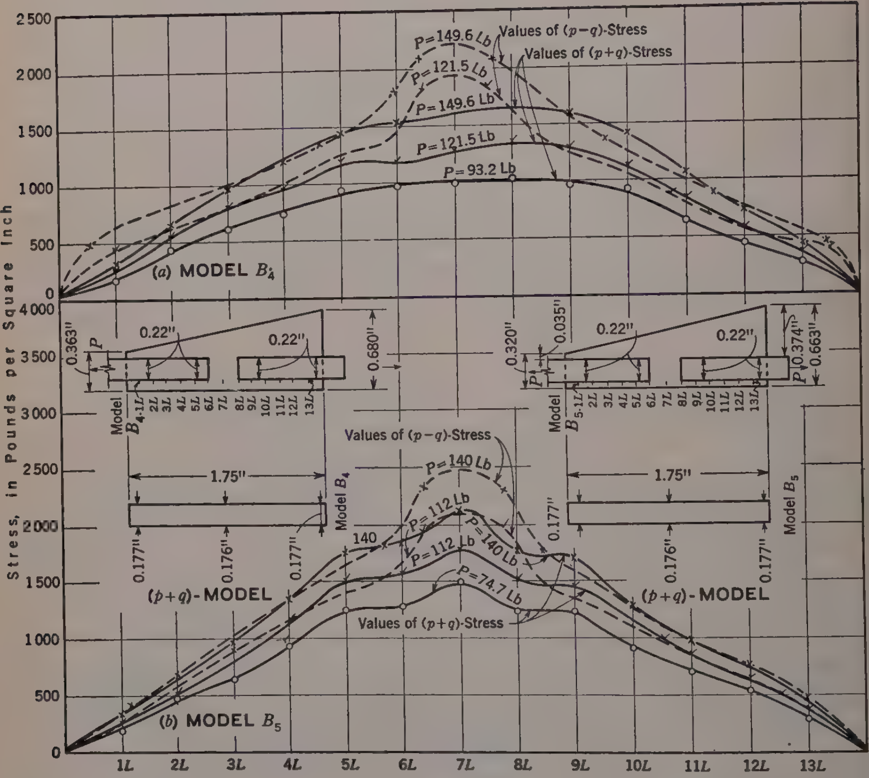

FIG. 9.—$(p - q)$ AND $(p + q)$ STRESS CURVES, MODELS B_4 AND B_5

The steepness of the curves at the ends of the plate shows the rapid building up of the stress in the plate at that point. The curves tend to flatten out near the mid-length of the legs, and from there on the $(p - q)$-curves build up again at the ends of the legs.

The secondary humps in Fig. 9 for Model B_5 at Point 9L are due to the stress (particularly the q-stress) tending to distribute itself over a greater part of the plate. There is considerable q principal stress between the ends of the legs in the plate; its tendency to spread over a greater part of the plate depresses the $(p + q)$-curves. No increment of $(p + q)$-stress is added in proceeding from Point 9L to Point 8L in Model B_5. The same phenomenon of $(p + q)$-stress distribution is shown also between Points $5L$ and $6L$ in the tendency of the $(p + q)$-curves to flatten out.

Sets of curves for $(p - q)$-stress and $(p + q)$-stress were plotted for the outer boundaries, with practical agreement. Since there is only one principal stress along these boundaries—the p-stress—the two corresponding curves for a particular model should agree. It was impossible to place the points of contact of the light interferometer exactly on the boundary. The contact points had to be moved in $\frac{1}{16}$ in. to prevent them from slipping off the model. This accounted for a slight difference in a comparison of a $(p + q)$-curve with a $(p - q)$-curve, which latter curve was plotted from fringe readings on the boundary. Since, for the most part, the p-stress has been the dominating stress, its direction is approximately parallel to the length of the model for most of the plate. The directions of principal stresses obtained from isoclinics, therefore, are not of great consequence for these tests.

Theoretical Consideration of Symmetrical Gusset-Plates

Many attempts have been made to find the theoretical stress distribution in symmetrical, riveted, gusset-plates, notably by the German and British investigators. Of these, one of the most recent and probably the most successful is C. Batho,[4] who assumes a uniform distribution of stress across the sections of the plates. This assumption may be in error as much as 20%, but the error of strain energy due to this uneven distribution is only a fraction of 1 per cent. Batho set up an equation for the energy in the entire symmetrical joint which adds the energy in the plate to that of the connecting members at the plate, and the energy in the rivets. The force on each rivet or row of rivets, perpendicular to the length of the member, is taken as unknown. Derivatives are taken of the resulting equation with respect to each variable force. To make the energy in the joint a minimum, these derived equations are equated equal to zero, and solved simultaneously for the forces at each rivet or row of rivets.

In order to verify the application of this theory to a practical case, the writer found some stress measurements between the rivets of a joint (see Fig. 10) which were made by C. Findeisen.[5] The full lines in the diagrams were plotted from Findeisen's measurements; the dotted graphs were plotted from results obtained by the application of Batho's theory by the writer to the joint tested by Findeisen. A fairly close agreement between the practical measurements and theoretical results is shown.

In a further effort to verify available theory for a symmetrical welded joint, an application of the formula for the distribution of shearing stresses in welded connections introduced by W. Hovgaard,[6] was made by the writer to the symmetrical bakelite model, A_1. Hovgaard's theory is based upon the principle of least energy, as is the theory of Batho for symmetrical riveted joints. Instead of deriving the work equation with respect to each force, X_n, on each row of

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[4] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structures Research Committee, pp. 114–127.

[5] "Versuche über die Beanspruchung in den Laschen eines gestossenen Flacheisens bei Verwendung zylindrischen Bolzen," von C. Findeisen, Verein deutscher Ingenieure, *Forschungsarbeiten*, Heft 229, Berlin, Germany.

[6] "A New Theory of the Distribution of Shearing Stresses in Riveted and Welded Connections," by W. Hovgaard, *Transactions*, Inst. of Naval Archts., March 26, 1931, pp. 108–126.

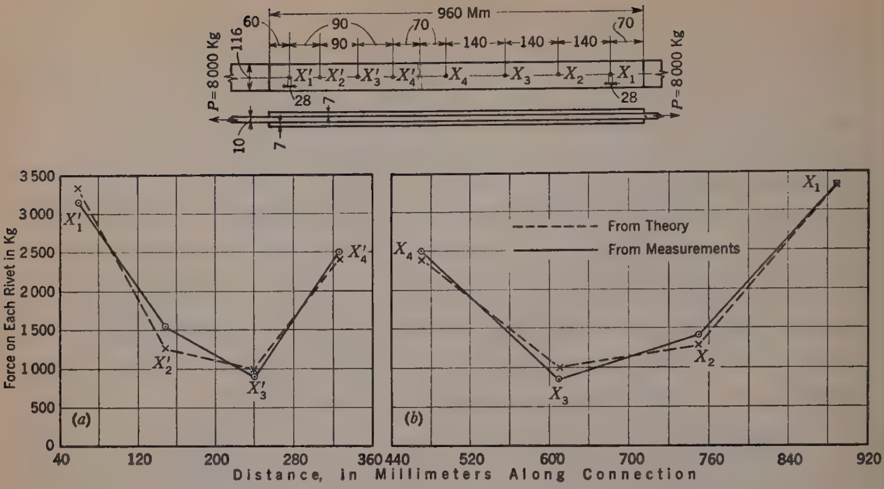

Fig. 10.—Batho's Theory Applied to Findeisen's Strain Measurements

rivets, and equating the derivatives equal to zero for a solution of least energy as Batho has done, Hovgaard writes an equation for the energy in the connection as a function of the shear stress at any point, X, of a welded connection of the member to the plate. The equation is minimized by variational calculus, and the solution of the differential equation obtained. The constants in the equation can be evaluated for a particular practical problem.

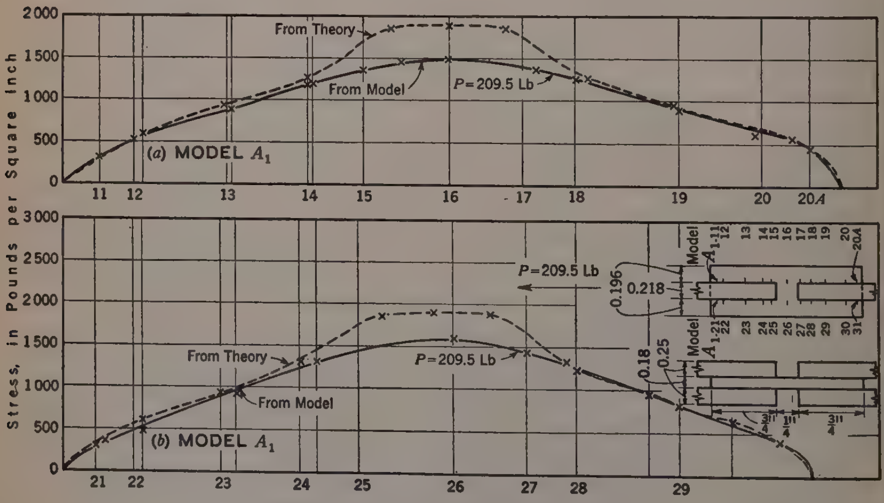

Fig. 11.—Hovgaard's Theory Applied to Model A_1

The results of Hovgaard's theory applied to Model A_1 are shown on the plotted graphs of Fig. 11. The full lines show the stresses along the legs in the plate of the model; the stresses from the equation applied to the model are shown by the dotted lines. From the ends of the plate for three-fourths of the distance along the legs, the curves agree very closely. Over this distance in the plate, one principal stress is much greater than the other principal stress. In fact, the smaller principal stress can be practically disregarded for this distance. The resultant stress is nearly equal to pure tension in the plate.

From the three-quarter point to the ends of the legs, the experimental curve leaves the theoretical curve for two reasons: (1) The smaller principal stress shown by experiment in Fig. 8 becomes greater, which indicates that the larger principal stress does not correspond to the exact tensile stress in the plate; and (2) there is a large increment of stress that extends from the end of the leg almost directly to the end of the other leg through the plate. Undoubtedly, the second is the most important reason for the deviation of the experimental curve from the theoretical. Path *b–b–b* shown in Fig. 12 is shorter and more direct than Path *a–a–a*; therefore, a definite increment of stress transfer takes place from the end of one leg directly to the end of the other leg. This increment relieves the transfer of stress from the side of the leg near the end, which is clearly shown by the deviation of the experimental curve from the theoretical one in Fig. 11.

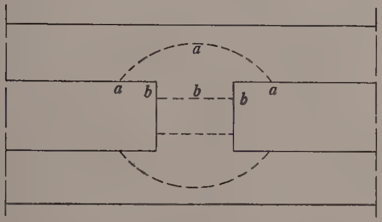

FIG. 12

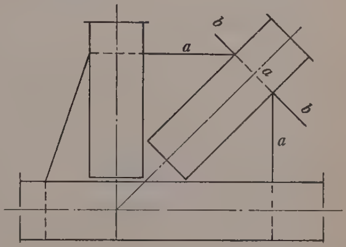

FIG. 13

RECOMMENDATIONS FOR THE SPECIFICATIONS AND DESIGN OF STEEL GUSSET-PLATES

There is a definite trend to-day toward the construction of larger steel structures with correspondingly larger gusset-plates. Sometimes, welds instead of rivets are used, but welds have not yet demonstrated completely that they are as reliable as rivets. Since the specification and design of gusset-plates have been the result largely of experience, general practise, and intuition on the part of the designer, an effort will be made to clarify and to extend the application of these known principles, as well as to add others based on the results of recent research.

A generalized stress solution for gusset-plates has not been found. In its present stage of development the Airy stress function does not offer much hope for the stress solution of gusset-plates. Likewise, there is no generalized stress solution about the phenomena of fatigue. Solutions for stress in symmetrical gusset-plates have been found both for riveted and for welded connections. The solution for symmetrical, riveted gusset-plates was found by Batho[4] whose solution is based on the principle of least energy, as is also that of Hovgaard[6] for symmetrical, riveted, and welded plates. The writer applied Batho's method to Findeisen's strain measurements on a symmetrical riveted plate connection; the writer also applied Hovgaard's method to the symmetrical bakelite model, A_1. The stress observations on bakelite models have been discussed in a previous part of this paper. The discussion will be broadened, however, to include considerations based upon general practice and present manufacturing procedure in order to give a more complete view of the art.

General Practice.—For small structures, gusset-plates can be designed and specified without great difficulty provided there is no reversal of stress. The rivet groups can be made comparatively short in the direction of the length of the member. Since there is less chance for distortion of the material, a shorter group allows a better distribution of stress between rivets. In small structures one or two gusset-plates are commonly used for each joint; but for larger ones, four or even more gusset-plates are necessary. The late J. A. L. Waddell,[7] Hon. M. Am. Soc. C. E., has recommended that there be as few cuts as possible in the plates, and that it is desirable to line up the gussets for appearance. For light gussets not subject to reversal of stress, they may be subordinated, without harm, to the appearance of the trusses and to fabrication economy; but for large and heavy structures the gusset-plates may well influence and even dominate some of the decisions as to the design and fabrication of the truss.

Long and narrow gusset-plates nearly always are computed, by elementary mechanics, more highly stressed than those with approximately equal axes. A long and narrow gusset-plate with diagonal members will have considerable eccentricity about the neutral axis of the gusset at some of its sections. Such connections are usually longer and less desirable than connections of diagonal web members with steeper slopes to the chord members. Long connections have a greater disproportion of stress among the rivets, with more low stressed rivets in the mid-length of the connections. An ideal gusset approaches a circle in shape, or a polygon of approximately equal axes, with all the forces intersecting at or near its center. A gusset-plate of such shape, due to its maximum width at the center, does not have great bending stresses. Furthermore, the longer section at the center allows the principle of least energy at the often critical center section of the gusset to function over a greater distance, with less stress concentration between the ends of the members (see Fig. 3). The shorter and wider rivet groups with respect to the length of the member tend to cover a larger rather than a smaller part of the gusset-plate, which implies that the gusset should be of no greater area than necessary. Gusset-plates with comparatively large unriveted areas increase the discontinuity which the joint represents.

The rivets should be of no greater spacing in a member connection than good design permits because the variation in stress from one rivet to the other will be less for the smaller spacing.

The cross-sectional area of the gusset where the member connects to it should tend to approach the area of the member at that point, and should not be (as is often the case) of much greater area. The plate shown in Fig. 13 should follow a line, *a–a–a*, rather than a line, *b–a–b*. The ideal of uniform stress transfer among the rivets can never be realized. The end rivets will always be the most highly stressed ones, as shown by the graphs of Fig. 10. The rapid building up of the stress in the plate, at the end of the plate, and at the ends of the legs of the models, is shown in Fig. 3. The stress difference between the end rivets and the intermediate ones can be reduced by making the rivet groups as short as practicable.

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[7] "Bridge Engineering," by J. A. L. Waddell, 1925, pp. 518–525.

Manufacture.—Gusset-plates should be manufactured with great care. This is especially true of the larger and larger gusset-plates that are continually being demanded. They should be made as nearly plane as possible with subsequent care in shipping and handling to insure the plane condition. Plates as thick as 1 in. cannot be placed in close contact even with rivets as large as $1\frac{1}{8}$ in. in diameter. Furthermore, even 0.75-in. plates cannot be fabricated under present practical conditions to insure contact everywhere. Some of the 1-in. plates in experiments reported by J. Montgomerie[8] were subjected to special care in fabrication. Since they were pressed together with high hydraulic pressure, the value of friction which they were able to develop was raised materially. Plates fabricated in the ordinary manner had a resistance to slip of 9 tons per sq in. of rivet area for thicknesses from 0.40 in. to 0.60 in.; 6.6 tons per sq in. for plates 0.75 in. thick; and, 4.3 tons per sq in. for plates 1 in. thick. After subjecting the 1-in. plates to hydraulic pressure and re-riveting them, the resistance to slip was raised from 4.3 to 7 tons per sq in. of rivet area. These experiments show the importance of avoiding the use of plates with contact surfaces that are not planes.

Painted or oiled plates are less resistant to slip than those with no foreign substance added. Better resistance to slip is provided in these cases when the line of stress transfer is perpendicular to the direction of rolling, rather than parallel to it. A well-driven rivet in single shear should provide a resistance to slip of 35% to 40% of the value of the tension in the rivet shank. This tensile value should equal the product of the area of the rivet by the stress of the rivet material at the elastic limit. Rivets driven at the temperature of about cherry red color, with a minimum clearance at the beginning of driving, provide the best resistance to slip. Lower temperatures in driving prevent excessive heating of the plates, which causes brittleness in some carbon steels and more especially in alloy steels. Alloy steels are usually specified with higher unit stresses, and have less resistance to slip than ordinary steels. Large slip resistance takes much load from the rivet hole-wall where fracture often occurs.

Careful fabrication and handling of the parts will add definite strength to the connection. The edges and faces of the plate should be free of notches and scratches where failure can begin. Plates of ordinary thickness, or thinner than 0.75 in., can be sub-punched and reamed, while plates more than 0.75 in. thick should be drilled and reamed. The reamer[9] should be heavy enough and of sufficient velocity to hold it in its true course. All burrs and bits of steel should be removed with care, and the holes at the pre-set end of the rivet should be counter-sunk nearly $\frac{1}{16}$ in., in order to help the flow of the molten metal of the rivet around this shoulder of the hole-wall. In the fabrication of parts, improperly matched holes are not justified.

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[8] *Transactions*, Inst. of Naval Archts., Vol. 65 (March 23, 1923), pp. 179–197, London, England.

[9] "Contribution à l'étude expérimental du Rivetage," par Paul Dugué, Bordeaux, France, 1927.

If an important gusset connection can be specified with plates of ordinary thickness and rivets of usual size, it is best to do so. The men who do the mechanical work are able to fabricate the materials of customary size with greater skill and achieve better results. Furthermore, most of the tools for fabrication of the parts are designed for average steel sections and rivets. If exceptional steel sections and rivets are specified, increments of improvement from skilled craftsmanship may be more difficult if not impossible to obtain.

Stresses.—The stresses in a plate with a re-entrant corner such as Point A, Fig. 14, are always greater than the elastic limit even with small forces acting on the gusset.[10] Sufficient material at Point A changes into the plastic state to provide a means of stress transfer around the re-entrant corner. If there happens to be reversal of stress, failure occurs at relatively low forces. From the standpoint of excessive stress, gusset-plates with re-entrant cuts should never be specified. If the highly undesirable, and seldom specified, case of a gusset-plate with a re-entrant corner is encountered, a fillet in the plate at the re-entrant corner, with as large a radius as practicable, will facilitate the stress transfer. It is nearly always possible to avoid gusset-plates with re-entrant corners, but the discontinuities in stress transfer at the rivets in gusset-plates cannot be avoided.

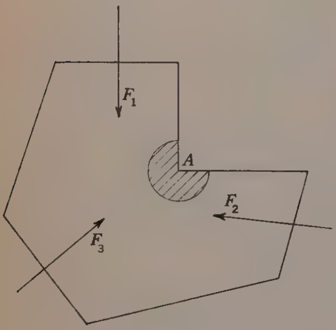

Fig. 14

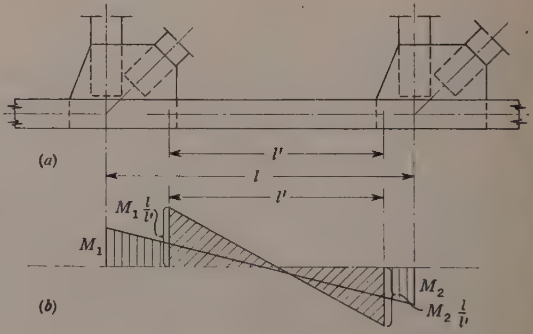

Fig. 15

Since the end rivets of a riveted connection are always the most highly stressed, it is of interest to consider their general method of stress transfer. If the section of the connecting member, carrying pure tension or compression, is equal to the section of the gusset, the percentage of total force taken by the end rivets varies in a manner similar to that shown in Fig. 10. If there are two rivets in a line the end rivets will take 50% of the forces. This percentage decreases to 36.6% for five rivets in a line.[11] From there on, for any addition to the number of rivets, the percentage of total force carried by the end rivet remains at about 36 per cent. Batho's data show that for the case in which the connecting member and the gusset-plate are equal in cross-section area, the end rivets cannot take less than 36% of the force on the member when the stress on them remains within the elastic limit. If there is no reversal, stressing the rivets beyond the elastic limit is of little consequence.

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[10] "Stress Distribution in a Re-entrant Corner," by J. H. A. Brahtz, A. P. M. 55-6, *Transactions*, A. S. M. E., 1933, Vol. 55, pp. 31–37.

[11] "Riveted and Bolted Connections," by C. Batho, First Rept. of the Steel Structures Research Committee, pp. 100–136.

When reversal occurs, and each stress passes beyond the slip-resistance capacity of the end rivets, the ultimate strength of the connection is greatly reduced. In the design of a riveted connection for reversal of stress, one of the stresses should be kept well within the frictional capacity of the rivets with special reference to the end rivets, to prevent the backward and forward motion in the rivet holes with resultant deterioration of the connection. The cooling of the rivet and the hole-wall after driving causes a lack of tightness, which can never be avoided with heated rivets.

It may be possible to design an important connection with rivets in double shear. This procedure will double the slip capacity of the rivets as well as the shear per rivet, and might justify the additional expense in the shop and in the field. This recommendation is given added weight by the fact that at a point on the surface of the plate into which the stress is transferred in single shear, the stress in the plate is twice the average across the section of the plate, with the stress on the far surface decreasing practically to zero. In order to keep one reversible stress within the friction capacity of the joint, it may be well to re-proportion the joint or even the structure.

Even for members without reversals, there should be as many end rivets in the connection of a member as possible; or, in other words, the number of lines parallel to the length of the member should be a practical maximum. The number of these lines may be increased by welding comparatively long lug angles to the sides of the member, thus adding two or even four lines of rivets to the connection. Since a welded connection is much stiffer than a riveted one, the objection to lug angles is largely overcome. No alternate rivets in the end rows of the connection should be omitted. W. M. Wilson, M. Am. Soc. C. E., with Messrs. J. Mather and C. O. Harris, reports[12] that the section of a tensile connection is not weakened nearly as much by specifying solid end rows of rivets as is generally believed; or, conversely, the section of the member is strengthened only a few per cent by omitting alternate rivets of the end rows.

The decrease in the number of rows shortens the length of the gusset-plate, and thus limits the discontinuity that it represents. Smaller gussets also increase the clear length between joints as shown by Fig. 15(*a*). Moments M_1 and M_2 are from secondary effects. When l' becomes greater Moments M_1 and M_2 become less as a result of the greater slenderness ratio of the member. A. Ostenfeld[13] states that the moments in the members at the gussets as shown in Fig. 15(*b*) have been increased in the ratio, $\frac{l}{l'}$, over what they would have been if the member had been free to bend through the entire distance, l. This factor, $\frac{l}{l'}$, is sometimes termed the gusset factor. In so far as the secondary stresses in a member are concerned, it is important to keep this factor as near unity as possible. From research by K. Havemann[14] the length, l', was found to extend somewhat into the gusset-plates, and he gives the formulas (see Fig. 16):

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[12] "Tests of Joints in Wide Plates," *Bulletin No. 239*, Univ. of Illinois, Urbana, Ill.

[13] "Teknisk Statik," A. Ostenfeld, II, p. 401, 1925.

$$y = a\,c^{x} \qquad (1)$$

and,

$$\log c = \frac{I_M}{I_G}\left(\frac{l'}{a}\left(\frac{I_G}{I_M}\right)\frac{a}{l'} - \frac{a}{l'}\right) \qquad (2)$$

in which a = distance from the old origin to the new X-axis, or the distance from the intersection of members to the end of the gusset; y = y-ordinate from the new X-axis, or the distance into the gusset from its end for l'; I_G = effective moment of inertia of the gusset-plate; I_M = moment of inertia of the member; and, $b' = a - y$. The end of the gusset denotes the end of the welds or the end row of rivets. Fig. 17 shows a gusset connection with distances, a, b', and l' of Havemann's formula.

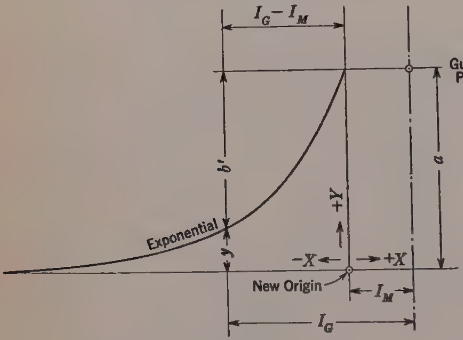

FIG. 16

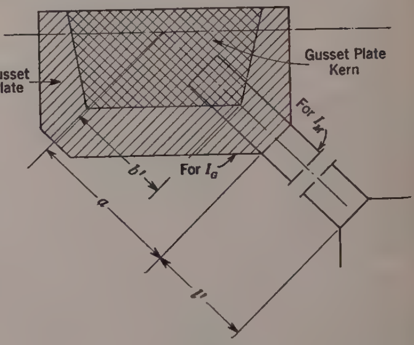

FIG. 17

Smaller gussets and lighter sections in the members of the trusses resulted from the multiplication of trusses in the bascule span of the Link Bridge recently completed on the Outer Drive in Chicago, Ill. The greater number of trusses with smaller gussets brought about greater lengths, l', between the gussets for secondary bending in the members (see Fig. 15). The lighter sections also increased the $\frac{l}{r}$ ratios of the members, with a corresponding decrease in secondary bending of the members at the gussets. The smaller gussets in area and section with their smaller rivet groups are better in this bridge than the larger gussets of heavier, fewer trusses would have been.

The secondary moments in the members at the gusset-plates are usually not large enough to be of great importance in the gusset-plate stresses, but with the shortening of the rivet groups the stresses in the end rivets due to these moments should be investigated. The shorter lever arms of the end rivets about the center of gravity of a shorter rivet group increase the load on the end rivets for a given moment. These end rivets are already the most highly loaded ones from the direct stress in the member. On a section of the plate that does not cut any of the members, the bending stress can be found roughly by the usual theory of flexure. This case is seldom met in practice, and may not occur at the critical section. Many gussets have been designed by cutting sections and examining the direct stress, moment, and shear, considering the rivets as uniformly stressed. The bending moment by this method is found by multiplying the forces of the members by their distances from the neutral axis of the section. This moment, divided by the section modulus of the plate at the section, and multiplied by the secant^2 of the angle made by the edge of the plate with the normal to the section, gives an estimation of the stress from bending, at the edge of the plate.[15] This procedure does not consider the non-uniform stresses in the rivets of connections as shown in Fig. 10, or the minimizing stress function shown graphically in Fig. 6(*c*).

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[14] "Beitrag zur Untersuchung ebener Fachwerke mit steifen Knotenpunkt Verbindungen," von K. Havemann, Darmstadt, 1928.

The model stress data all show the non-uniform stress transfer into the plates along the legs of the models, with the greatest shearing stress transfer at the ends of the plates and the ends of the legs. Fig. 3 also shows the tendency for concentration of stress in the direct path between the model legs from the principle of least energy. The highest $(p - q)$-fringes shift toward the narrow side of the plate in the unsymmetrical plates, or in the case of Model D_3 toward the side of the plate with least area. If the unsupported edge of a steel gusset-plate is stressed in compression, the edge will buckle before failure, throwing more moment and direct stress into the interior of the plate. Parts of the gusset-plate in direct line between members should be designed with the object of providing for all these additional stress increments. The connections of the members to the plate should be made wide, with as many lines of rivets for a riveted connection as practicable.

ACKNOWLEDGMENT

The work[16] reported herein was made possible by the varied facilities at the University of Michigan. The writer is especially indebted to Robert H. Sherlock, M. Am. Soc. C. E., for encouragement and assistance in this research. All the tests were made at the University of Michigan except those of Model *D*, which were made at the Armour Institute of Technology, Chicago, Ill.

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[15] "Bridge Engineering," by J. A. L. Waddell, 1925, pp. 519–529.

[16] This paper is a part of a thesis submitted by the writer for a D.Sc. degree at the Univ. of Michigan. A complete copy of the thesis has been filed for reference in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STATE-WIDE SURVEYING PRACTICE IN MASSACHUSETTS

A SYMPOSIUM

PAGE
Establishing a System of Rectangular Co-Ordinates.
By Elmer C. Houdlette, Esq. 1848

The Land Court and Its Engineering Procedure.
By Clarence B. Humphrey, Esq. 1868

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ESTABLISHING A SYSTEM OF RECTANGULAR CO-ORDINATES

By Elmer C. Houdlette,[1] Esq.

Synopsis

The story of the Massachusetts Geodetic Survey is that of the matching of two needs with one opportunity. This is not in its generality so very different from the ideal that underlies, in theory, the pattern of all projects to which the Federal Government lent support. Within the province of engineering there are certain special areas of knowledge in which advance has been halted and is awaiting some outside form of impulse. Endowed research may approach them, or a governmental agency, acting in the general interest, may sometimes undertake them; but in the main, they are not employment producing under ordinary conditions.

The Commonwealth of Massachusetts felt that there was a great need for a State-wide local control survey and sought to link it to a confronting human need.

Control surveys are necessary prerequisites to highway planning, Land Court surveys, map-making, flood control, water supply, sewage disposal, and perpetuation of private and public boundaries. They insure the accuracy of measurements incident to such projects, reduce the cost of maintaining the required degree of accuracy and provide means for co-ordinating adjacent projects executed concurrently or at different times.

Faithful use of the fixed points established by the Massachusetts Geodetic Survey will eliminate, for all time, the multiplicity of origins, the over-supply of level datum planes, and the stupendous economic loss of thousands of uncontrolled surveys.

Introduction

The representation of the face of Massachusetts, like the photography and portraiture of the face of Man, has undergone, in the course of the decades, certain easily discernible changes. Due partly to a growing naturalistic sense, partly to improved means of reproduction, it has all been in the direction of a more faithful copy. The story of these changes is one of gradual lifting of that face from its vague and indistinct contours to the more evenly defined surface recognized to-day.

The practice of accurate, up-to-the-minute map-making, courted by, and even indissolubly married to, highways, railroads, regional promotional enterprises, and travel bureaus—as well as sponsored by its traditional guardians, the Federal, State, and local governments—is subordinate to and consequent upon extended travel, broadened communications, and increased land use. As such the demands upon map construction through the years have been conditioned upon the size of those developments and the growth of the science of cartography itself. Its rise has been slow, steady, and rewarding.

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This is no less true of Massachusetts than of other regions. Thus, in 1829, the necessary preliminary step was taken by the Massachusetts House of Representatives, which then adopted an order appointing a committee "to consider the expediency of determining a map or maps of this Commonwealth as the public exigency requires and report thereon to the next session of the Legislature." Thus, having presumably established the necessity for such a map, the Legislature, on March 1, 1830, passed a resolve directing each city, town, and district to make a plan of its territory on a scale of 100 rods to an inch and to file such plan with the Secretary of the State before June 1, 1830.

Likewise, as a co-ordinating effort in the same direction, on March 30 of the same year, the Legislature directed the Governor to appoint a surveyor to make a skeleton map of the State, the intention being to use this survey as a basis for plotting the information furnished by the various towns. Accordingly, two months later, Robert Treat Paine, of Boston, Mass., was appointed as Chief Engineer, and James Stevens, of Newport, R. I., as an Assistant; Mr. Stevens undertook the triangulation and Mr. Paine the astronomical work necessary to determine the latitude and longitude of many points in the State. Simeon D. Borden, a mechanic employed by the Pocasset Manufacturing Company, of Fall River, Mass., designed and constructed a base-line measuring apparatus. He was persuaded to resign his position and undertake the measurement of a base line in Massachusetts. This line, 39 009.73 ft long, was laid out on the plain west of the Connecticut River in the Towns of Hatfield, Whately, and Deerfield. From it was developed a network of triangulation containing about 550 stations and these data were used to provide an outline map of the State, plotted on a polyconic projection and divided into five sections. In each section a central meridian was determined and from it rectangular co-ordinates were computed.

All town maps were reduced to a common scale and an attempt was made to co-ordinate them, along with the State lines and triangulation points, into a skeleton map that would use all data available at the time. In 1841, the map was completed and engraved in six plates; it was largely a one-man affair in its later stages by virtue of Mr. Borden's own re-surveying and fudging. In the end this map proved to be too thickly garbed with inaccuracies and mis-scalings to serve the increasing demands.

Beginning in 1844, the United States Coast Survey measured a base line along the railroad tangent of the Boston and Providence Railroad, running through the Towns of Attleboro and Mansfield and ending in the southeastern part of Foxboro. Upon this base line (10.75 miles long) there was developed a net of first-order triangulation as part of the general system of the eastern oblique arc, extending from the Epping Base, in Maine, through this Massachusetts Base, and thence to the Fire Island Base, in New York State. The precise stations of the Borden survey were included and adjusted as part of this general plan.

Under Chapter 72 of the Acts of 1884, the Massachusetts Legislature established the Massachusetts Topographical Survey Commission, consisting of three members. This body was delegated to co-operate with the U. S. Coast Survey in completing the necessary control for the preparation of a contour topographical survey and map of the Commonwealth. Later, under the Acts of 1888, Chapter 336, the duties of this Commission were broadened to provide for defining and preserving town boundary lines.

During the period, 1898–1900, the Topographical Survey Commission completed its assignment and published forty-two atlases for as many cities and towns in the eastern part of the State. With this task completed the Commission was dissolved and its duties transferred to the Harbor and Land Commission, which, in an effort to complete the assignment at cheaper cost, combined adjacent towns in one atlas.

By the end of 1912 all the atlases had been published for all the cities and towns east of the Connecticut River. During the period, 1913–1915, the atlases of the section west of the Connecticut River to the New York State line were published.

NEED OF CONTROL

However, in spite of the realization of this long-brewing task, the job, so to speak, was just beginning. Every one was still "in the dark" as to the exact location of a piece of property. True, there were maps showing every city and town in the State; true, by virtue of these maps, progress along the road of charting the State had been furthered; but people were still floundering in uncertainty.

They were still using assessors' maps based upon low standards of control and loose adjustment. The data for such maps had been acquired by closed traverses, supplemented by disconnected traverses, into which were fitted old surveys as described in deeds. These descriptions were rarely confirmed by a map, and no check had been made of the area said to be included by the bounds as given. Many towns still had failed to set the widths of their most important thoroughfares, thus creating a state of confusion for abutting owners who desired to make improvements on their land.

Engineers still were using about twenty-five or thirty different datum planes for level work throughout the State. Suggestive of the diversity that has existed is the variety of the following specimens: Mean low water at Boston Harbor; mean low water at Commonwealth Pier; Boston low-water datum; and mean tide level, Boston City Base, and Metropolitan Water-Works Base. In addition, there are many others.

The economic loss of thousands of uncontrolled transit surveys of all kinds was running into millions of dollars for the State as a whole. Such surveys were based on an unlimited assortment of assumed, magnetic, or true meridians. Plane co-ordinates, if used at all, were referred to an equally unlimited number of origins without regard of their geodetic positions. Levels were based on every conceivable kind of assumed datum plane. It all represented a mass of the wildest confusion.

VALUE OF CONTROL

How much more practicable and scientific, indeed, is a system of uniform and universal measurements used by and serving one and all communities of the State? There is a "hoary-haired truth" that runs to this effect, "You may get out of something what you put into it." This is particularly true of the earth. As the yield of a spring's plowing and sowing, and of a summer's care, protection, and re-plowing, is a fall crop paying triple or more the effort that went into it, so the yield of properly manipulated land control is a saving in money, time, and labor. Not only is this true, but it is also a fact that data gathered from controlled surveys afford a distinct advantage to those who come after.

Engineers are familiar with the advantages of geodetic co-ordination and through them all surveys are made permanent and indestructible, and are incorporated into one uniform system in which every survey is properly placed and related to all others. A controlled survey, being permanently fixed, may be drawn upon at any future time for many other purposes aside from that for which it was originally intended.

The first effort to place elevations in Massachusetts on a mean sea-level datum by means of spirit-leveling, was made under the auspices of the Massachusetts Topographical Commission in 1893. The late C. H. Van Orden, M. Am. Soc. C. E., an officer of the U. S. Coast and Geodetic Survey, was engaged to extend a line of precise levels from Boston, westward across Massachusetts into New York State, to a connection with the precise leveling which had been previously run along the east side of the Hudson River. His data,[2] published in 1894, included a detailed statement concerning instrumental equipment and field methods. In 1923 and 1927, several lines of precise levels were run by the U. S. Coast and Geodetic Survey in Massachusetts and, finally, in 1933, this organization re-ran the Van Orden line from Springfield, Mass., to Albany, N. Y.

In November, 1929, finally persuaded as to the importance of a mean sea-level datum for all surveys, the State Department of Public Works authorized the purchase of a precise level and two invar-faced rods. This marked a major and propitious development in the slow progress that was being made toward the adoption of mean sea-level datum for the Commonwealth as a unit.

Previous to the adoption of the Lambert projection, the increasing use by the Department of Public Works of the twelve Land Court systems of co-ordinates developed a need for simplification. Through conferences and co-operation between the Land Court and the Department of Public Works, on April 1, 1933, geodetic control was established on eight co-ordinate systems, six of which remained as before, while two systems replaced four and two were discontinued. As the need for trunk-line highways passing through many towns and counties increased, the use of eight origins became more or less complicated in that equations of co-ordinates and azimuths had to be applied every time a survey passed from one zone to another. This confusing situation has been clarified and difficulties incident to practical application have been greatly simplified by local control, with ample bench-marks, upon one standard basis.

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[2] Rept., Commrs. of the Topographical Survey of Massachusetts, 1894.

PROJECTION SYSTEMS

Two methods of projection offer themselves for selection, the choice depending on the geographic figure of the State. A State with a narrow east-west width can be completely shown on a transverse Mercator projection, while another State with a narrow north-south width can be completely shown on a Lambert projection.

In the transverse Mercator projection, standard small circles are selected equi-distant from the meridian of longitude passing through the center of the figure. The development of the plane of projection and the transfer of terrestrial points to the plane is similar to that pursued in the Lambert conformal conic projection, to be described subsequently. Both methods approximately attain preservation of angles at a small sacrifice of accuracy in scale, provided the representation does not go beyond the critical dimension of the projection. The choice of projection depends upon the relative geographic dimensions of the area and the error in scale to be accepted.

Since the maximum error of closure accepted in second-order traverse is 1 : 10 000, it is natural to adopt this criterion as the maximum error in scale acceptable for distances on the projection. Applying this criterion it is found that the ratio will not be exceeded on a Lambert conformal conic projection when the north-south distance of the area to be represented does not exceed 158 miles. Likewise, on a transverse Mercator projection, the east-west dimension must also not exceed 158 miles. As the critical dimension is exceeded, the error in scale increases rapidly. If both dimensions of the area to be mapped exceed the critical dimension then two or more projections will be required to hold the scale ratio to an acceptable limit.

The adoption of either of these methods of projection provides a single system of plane co-ordinates for the entire area. After the geographic co-ordinates of the triangulation stations are converted to plane co-ordinates the computations to determine the position of all intermediate points established by traverse is simplified without any sacrifice in accuracy.

Previous limitations in the form of required use of geodetic formulas and methods, an impracticable expense in many cases, have been eliminated since the formulation of the State plane co-ordinate systems by the U. S. Coast and Geodetic Survey. Any engineer or surveyor in Massachusetts can now utilize the plane co-ordinate system of the Lambert conformal conic projection, and, at the same time, can conduct his work according to the standard methods of ordinary plane surveying.

DIFFICULTIES IN USING SPHERICAL CO-ORDINATES

As is well known the surface of the earth is not a plane but a curved and irregular surface. For purposes of surveying and mapping, however, an ideal surface is assumed which approaches very nearly the shape of the earth if the entire surface were composed of water. This assumed shape is not a sphere, but an ellipsoid of revolution.

Geodetic surveying comprises the extensive work of making observations and computations of positions on the surface of this assumed spheroid. It becomes extremely complicated when the surveys extend over large areas. Also, it requires an intimate knowledge of higher mathematics and astronomy, and the highest degree of precision in the field observations, using the most elaborate and expensive field equipment. All geodetic stations on the earth are expressed in terms of latitude and longitude; that is, each one is referred to a definite place in the network of meridians and parallels that are assumed to cover the surface of the earth in a definite and standardized manner.

It must be perfectly evident that geodetic surveying, although absolutely essential as a basis for precise control, is impracticable for use in ordinary survey work due to the rigorous technique and costly equipment required. Perhaps, for this reason, in spite of the fact that it reached a high stage of perfection and was taught in technical schools many years ago, engineers as a whole have not taken an extensive interest in the general subject, and, consequently, have not developed to the full any standard of practice which might enable them to utilize economically the great inherent possibilities to be found in the existing surveys of the U. S. Coast and Geodetic Survey.

Within these surveys lies a wealth of data, which can be made of tremendous value, not only to engineers and surveyors in every field of activity, but through them to every corporation, commonwealth, county, municipality, property owner, or individual who pays for any kind of instrumental surveying. This value lies in the use of the monumented stations of the national triangulation system as starting and ending points controlling all surveys. Since these points are accurately located on the spheroid, and in relation to one another, they furnish a means of checking the work and of adjusting the errors in the survey. The system of plane co-ordinates permits the use of these stations for control purposes without involving the complicated computations otherwise necessary. In a word, it constitutes a mathematically sound and accurate method of transposing work from geodetic or spherical to plane surveying methods.

The Plane Co-Ordinate System as Adopted for Massachusetts

The Lambert projection adopted for spreading local control stations throughout Massachusetts is based on a geographical frame selected with particular reference to the shape and area of the State. The frame consists of two standard parallels of latitude, held true to scale, and a central meridian of longitude. The north-south dimension of Continental Massachusetts is considerably less than the critical, with consequent reduction of error in scale distances.

In order that the scale ratio applied to distances at the middle of the State shall be equivalent, although of opposite sign, to that for the northern and southern boundaries, it is necessary to select the two standard parallels of latitude at distances from the most northerly and southerly points approximately equal to one-sixth of the north-south dimension of the State. Consequently, the north latitudes of 41° 43′ and 42° 41′ were selected, together with the meridian of west longitude of 71° 30′.

The geographic frame for the insular territory was similarly selected. The standard parallels of 41° 71′ and 41° 29′, together with the meridian of 70° 30′, form the frame. The latter projection excellently illustrates the decrease in scale ratio which accompanies a decrease in the area mapped.

The plane of projection is formed by a cone of revolution intercepting the earth's surface along the two standard parallels of latitude chosen for the frame. The apex of the cone is on the axis of the earth projected through the North Pole into space. The imaginary cone of projection passes beneath the earth's surface in the area between the parallels and above the earth's surface north and south of that area. Approximately two-thirds of the distance between the extreme northerly and southerly continental points in the State lie between the parallels. The remainder is equally divided and lies outside.

Distances between points on the plane between the standard parallels are shorter than those between the same points on the surface, while outside the standard parallels the reverse would hold. Only distances between points along the two standard parallels are true distances and, therefore, equal upon both the plane of projection and upon the surface of the earth. The angles between lines connecting points upon the plane are approximately equal to the angles between the lines connecting the identical points on the earth's surface. This equality of angles is denoted as "conformal."

The U. S. Coast and Geodetic Survey selected the State projection and prepared the plane co-ordinate projection tables to facilitate the computations necessary to transfer a point on the earth's surface, defined by geographic co-ordinates, to an equivalent position on the plane of projection defined by numerical co-ordinates developed on the single State system. The table for changes in latitude gives the numerical value of the radius and Y-co-ordinate on the plane of projection for each minute of change, together with the tabular difference per second for change in the Y-co-ordinate and correction for scale, with their respective logarithmic expressions. The table for changes in longitude gives the change in the divergent angle for each minute. These tables and an acceptable manual of logarithms, expressed to eight decimal places, equip the computer to convert geographic co-ordinates to plane co-ordinates.

The meridian through the origin might be used as the prime meridian, but to reduce the mechanical labor in applying interpolations in logarithmic tables it will facilitate computations to select, in this case, the meridian of west longitude, 71° 30′, just east of the central point of the State, and assign to it the X-co-ordinate value of 600 000 ft, this places the origin on a meridian west of the most westerly limits of the State. Selecting as the other base the parallel of north latitude of 41° 00′ (south of the most southerly point), places the origin of co-ordinates for the single State system at the intersection of these two lines.

History of the Massachusetts Geodetic Survey

The organization entrusted with the task of applying the Lambert conformal conic projection to Massachusetts is known as the Massachusetts Geodetic Survey. Established in November, 1933, as the Massachusetts Local Control Survey, its duty was primarily to make more widely accessible to local engineers the advantages of the Federal triangulation net already laid in the State. In general, its goal was to lessen throughout the State the distances between the marked points in the net by setting down intermediate points. At that time the spacing between marked points, approximately 25 miles apart, giving only $12\frac{1}{2}$ miles from any point to a triangulation point, was too wide for effective local use. The task and expense of running a 12-mile traverse or line of levels to the site of a project were more than burdensome for an engineer undertaking a small job.

The Massachusetts Local Control Survey began its assignment with the running of traverse and lines of levels along highways and railroads from one triangulation station or bench-mark to another, with monuments set at intervals of about two miles en route. The accuracy of the work was to be such as was appropriate to the short length of line and as could reasonably be attained with the instruments available. The error of closure allowed for traverse was 1 : 10 000; and for levels, it was according to the formula:

$$e = 0.4\,\sqrt{s} \qquad \text{(1)}$$

in which s is the distance, in miles. The plan of operation was as follows:

(1) The U. S. Coast and Geodetic Survey created a unit of its own trained personnel to have administrative and technical jurisdiction over the work.

(2) It appointed an engineer as Director of the work in each State.

(3) Through the State Director, it obtained the loan of surveying instruments, office space, drafting-tables, and other necessary equipment.

(4) The State Director closely supervised the surveying operations to assure economy and efficiency of performance and mathematical accuracy of results, in order that the prescribed standards of the U. S. Coast and Geodetic Survey might be met fully.

(5) The Director established a central office where the results of the field work could be adjusted and computed as the work progressed.

The early period of the project was attended with many of the uncertainties of adjustment and operation, characteristic of the emergency undertakings of the Government since the depression. The first limitation came in less than two months after operations had been begun. With thirty parties in the field in January, 1934, an executive order, reporting the insufficiency of funds to carry on the entire CWA (Civil Works Administration) project, called for a reduction of the personnel from 375 to 75 workers, by February 15. Accordingly, the field equipment was collected preparatory to tying up the project altogether.

Up to this time the office work consisted chiefly in computing and plotting on the Lambert projection the co-ordinates of the corners and mid-points of the fifty-four topographical sheets of the State for the Massachusetts Land Court. However, on February 23, efforts to have the project personnel approved for 182 employees were successful. Fourteen field parties and forty-one office men were re-hired forthwith; but in spite of this reinforcement the force was still insufficient to continue the traverse work west of the Connecticut River. This plan, for the time being at least, had to be abandoned.

The next major adjustment came on April 4, 1934, when the project was transferred to the ERA (Emergency Relief Administration), whose original approval of 177 workers was increased by August to 230. This provided twenty-one field parties. An order of September 24, 1934, reducing all salaries to a common basis of $16.80 per week, from Supervisor to Rodman, was rescinded within a week.

The first few months of this re-adjustment brought along anxiety and disturbing jolts. The accompanying loss in personnel definitely tended to obstruct the program. However, the spirit was unflagging and certain accomplishments stood out from the condition of disruption as indicated by the report for the year ending December 31, 1934:

(1) The project had completed 585 miles of traverse, including 1 170 monumented stations; and 343 miles of preliminary computations showed an error of approximately 1 : 18 400.

(2) The project had completed 304 miles of first and second-order levels, including the establishment of 679 bench-marks.

Meanwhile, by virtue of the triangulation and projection work, the demand for added service was increasing. One valuable use pointed in the direction of improved forest fire control. It happened that up to this time the "tying-in" of fire towers with triangulation was not complete. The Department of Conservation in the fall of 1934 requested the Survey to complete the "tying-in" process; likewise, in order that the Lambert projection might be used on the topographical sheets for the location of forest fires, it desired that the Lambert co-ordinates be computed for all the fire towers in the State. At about the same time the Massachusetts Land Court requested a computation of the Lambert co-ordinates for all the triangulation stations on Martha's Vineyard and Nantucket, as the triangulation stations on these two islands had been adjusted on the 1927 North American datum.

At the beginning of this project, the geographic positions of all the established triangulation stations in the State, with the exception of the seventeen precise stations, were based on the North American datum. Until such time as the remaining triangulation stations were either re-computed or re-adjusted on the 1927 North American datum it was necessary, for preliminary traverse computations, to apply to each triangulation station as it was used, a correction to the published position to place it upon this datum. The rapid determination of the amount of correction to be applied to a given point was facilitated by the construction of a correction chart based upon the seventeen precise stations furnished by the U. S. Coast and Geodetic Survey.

These stations were plotted in correct position on an accurate map of the State and the amounts of the differences in latitude and longitude between the North American datum and the 1927 North American datum were noted. These points were connected by direction lines which were divided into equivalent parts representing the units of difference in latitude between the stations. These subdivisions established points of equal correction which could be connected to equivalent points on adjacent direction lines. The space between the connecting lines defined an area within which all stations were to receive equivalent correction in latitude. The distribution of the corrections in longitude were similarly determined.

Thus, the map of the State was covered with a somewhat rectangular and more or less regular grid, the unit subdivisions being about a mile apart in the east and west direction and somewhat less north and south, corresponding to connections to the nearest 0.001 sec in longitude and latitude. This map became a chart showing the amount of correction to be applied to the geographical co-ordinates of points included between the parallels.

Plotting the position of a point (defined by geographical co-ordinates on the North American datum) places it in one of the numerous grid areas. Thus, the corresponding amount of correction is determined and the position of the point is adjusted to the 1927 North American datum.

This chart was found, by later triangulation, to give results in general only to the nearest foot, but it served the purpose of determining a preliminary adjustment of all the traverses and showed whether the work had been completed in accordance with the specifications of second-order accuracy. Its main purpose, however, was to give a quick solution of control points in order that this information might be available to the United States Geological Survey in re-mapping the State.

In 1935 the equipment and personnel of the project were strengthened to the extent that the original program could be more fully and thoroughly completed and new work could be handled in its course. In January, the quota of employees was increased to 300, including 26 field parties.

A further increase in personnel to a total of 367 was made upon the transfer on November 1, 1935, of this project from ERA to WPA (Works Progress Administration) control. However, the accompanying shortening of working time from 35 to 24 hr per week brought about a decided slowing down of the work which was ever increasing in volume. Prior to January, 1936, transits and levels used in all field work were loans to the organization, and, in general, they were only in fair condition. As the transits read merely to the nearest minute, about 15% of the traverses had to be rejected. In time, this state of affairs was brought to the attention of the local officials who obligingly agreed to the purchase on a rental basis, of twenty-four new 20″ and 30″ transits. With this improvement, errors of closure approached that of first-order accuracy and rejections were reduced to about 5 per cent.

Additional undertakings by the Survey call for special mention. During 1935 it started preparing maps for each city and town in the Commonwealth, showing the location of all traverse and triangulation stations and bench-marks, such maps to be used by the localities for local control surveys within their limits. In October, the project was requested by the Department of Public Health and the various municipal water supplies to establish bench-marks, determine the heights of high and low water, the crest of dams and spillways, and establish the elevation of stand-pipes and of the ground-water level in driven wells on a common mean sea level basis. Such a unified survey makes it possible to judge whether there is a presumption that a given city or town might find it physically feasible to obtain additional water from a neighboring source.

All systematic and reliable information about relative levels is certain to help in planning. It will be of potential assistance in times of emergency, which can partly be foreseen as to type but which can never be foreseen as to time or detail. There is a natural community of interest between adjoining towns and cities because of the reciprocal assistance they might afford each other in times of need, when one had been better able than the other to cope with drought conditions. Less spectacular is the opportunity for temporary tie-in with a neighboring supply in the event of temporary breaks or of breakdown of the servicing. A more glaring illustration is the assistance that sometimes can be rendered by a neighboring community in the event of large or protracted fire, or of suddenly threatening depletion of the ordinarily sufficient reserve because of fire.

During the same year (1935) requests for control were received from sixteen cities and towns around Boston. An additional request was received for a large amount of control for military purposes within the area of the newly established National Guard Camp at Bourne on Cape Cod. One encouraging feature about these requests was the marked inclination toward the adoption of the State co-ordinate system. By the end of that year 1 618 miles of first and second-order levels had been adjusted and 1 275 miles of traverse completed in the field with an average of error of closure of 1 : 23 100.

The supreme test of the strength and resources of the organization and of the fortitude of its engineers presented itself in March, 1936, when large areas of the State were engulfed by unprecedented conditions of flood. The Massachusetts Geodetic Survey was called upon to accept a task never before undertaken, namely, the collection, by specially directed and equipped eye-witnesses, of authentic high-water data. A plan to establish a systematic series of bench-marks at bridges and dams along the principal rivers in the State already had been partly executed. Thus, with a trained organization and a substantial vertical control base achieved, all was in readiness to launch the flood survey; and on March 18, supervisors were instructed to notify all party chiefs to prepare their men for quick action in the field, if and when the emergency arose.

The men were to reach the flooded area over as many roads as possible, in order to place the high-water marks at well distributed points along the entire course of streams where flooded conditions prevailed. An order was issued on March 19, and the full personnel of about 200 field men were thus assigned to placing suitable marks, recording high water at the site and at the very time it occurred. This work continued until the extreme flood stage had passed. As rapidly as the field reports were received at the Boston Office, the position of each mark was plotted on the town maps and these reports were grouped and dispatched to the several field supervisors for allotment to especially organized level parties.

During the fall of 1936, the U. S. Coast and Geodetic Survey asked the State Survey to determine the magnetic declination at about seventy stations scattered over the State, from the tip of Cape Cod to the New York State line, including all the islands. The results of 24-hr observations made at Mt. Wachusett for seven consecutive days, to determine diurnal variation, were compared with the observations at the Observatory in Cheltenham, Md.

In the beginning of 1937, also by request of the U. S. Coast and Geodetic Survey, two first-order arcs of triangulation across Massachusetts were started, together with the necessary second-order triangulation, to tie in all the completed traverses, in order that a final adjustment might be made on the 1927 North American datum.

An idea of the amount of Survey work accomplished may be gained from the following summary, through September, 1938:

Traverse:	
Extent, in miles	2 605
Number of stations established	4 700
Number of stations with preliminary co-ordinates	3 427
Number of stations with final co-ordinates	1 272
Average error of closure	1 : 35 400
Triangulation:	
Number of stations established	550
Number of first-order adjusted stations	55
Number of second-order adjusted stations	223
Bench-Marks:	
Extent of first-order levels, in miles	1 320
Extent of second-order levels, in miles	3 760
Number of first-order bench-marks	1 850
Number of second-order bench-marks	8 101
Magnetic Stations:	
Number of stations established and adjusted	68
City and Town Maps:	
Number of maps completed	385

Publication Program

One interesting feature of the Survey's work was the opportunity of printing its records, investigations, and standards. In June, 1935, the first publication of the Survey entitled "Leveling in Massachusetts" was issued. This volume contained the descriptions and elevations of all bench-marks, including those established by the U. S. Coast and Geodetic Survey and by the State project, together with a history of leveling in Massachusetts, specifications for first and second-order leveling, a discussion of the various datum planes, and a chart showing the relation of some of the principal datums in use and their relation to mean sea level.

Fortunately, the Traffic Survey was in the same building as the Survey. As its work was about to be terminated in August, the Geodetic Survey fell heir to a complete plant with all its divisions, including reproduction equipment, modern multilith and mimeograph machines, photostatic and reproducing cameras, dark room, and plate-making equipment.

Immediately the publication of a second volume entitled "Computations of Spherical and Grid Coordinates with Tables" was undertaken. This book contained instructions for solving geodetic positions, a description of the Lambert projection, and directions for computing co-ordinates, together with tables.

During the year, a request was received from the U. S. Geological Survey to undertake as part of the survey work in this State, the determination of the elevations above mean sea level datum, of the reference marks at the forty-two stream-gaging stations. In 1937, a report was issued entitled "Stream Gaging Stations in Massachusetts." This report contained notes on equipment and operation of stream-gaging stations, and stream-flow data as compiled by the U. S. Geological Survey, together with the descriptions and elevations of the reference marks at each of the forty-two stations.

Information gathered out of the flood-control activities formed one of several very significant books published by the project during 1936, "High Water Data: Flood of March 1936." This is a tome of about 350 pages, based upon about 1 400 observed points which required about 800 miles of levels to determine the elevations.

An April, 1936, release by the project was a 46-page, very readable book, entitled "Status of Land Surveying in Eastern Massachusetts." Another volume, "Technical Procedure for Geodetic Surveys," released the following month offered a manual of instruction in instrument operation, methods of procedure, and recording of data necessary for standard triangulation, traverse and leveling survey work. This publication has proved of immense value in standardizing survey work in the State. A year later a volume was published entitled, "Plane Coordinates of Minute Intersections of Meridians and Parallels of Latitude Based on the Massachusetts State System."

The various volumes mimeographed by the Reproduction Section testify to painstaking work on the part of the publishing unit of the project. The demand for these publications was more than flattering, especially in view of the lack of any provision for advertising or book promotion campaigns.

The Usefulness of the Plane Co-Ordinate System

What is the procedure of a surveyor who undertakes to make use of the plane co-ordinate system as devised for Massachusetts? He will have previously obtained from the agency the plane co-ordinates of the monuments at which he proposes to begin and to end his survey, and also the plane bearings or the plane azimuths, from these monuments to their respective azimuth marks. Then, setting up over the first monument with the vernier at zero, he sights to its azimuth mark. From this line of sight he turns the angle and measures the distance to the next point in his survey which the instrument is to occupy. He also computes the plane bearing of this line from the given plane bearing between monument and azimuth mark. He carries his calculated plane bearings, in the usual manner, right on to the end of the survey at the terminating monument, and on through the monument to its azimuth mark. Instantly, he has a check on all the instrument work, for the calculated bearing from monument to azimuth mark should be the same as that which was furnished by the agency.

After distributing his angular error, he computes the plane co-ordinates (latitudes and departures) of his survey, starting with those furnished him for the beginning monument. His co-ordinates for the terminating monument should agree with those previously furnished him. Thus, he not only has a check on both instrument work and chaining, but he has made a controlled survey which is permanent, and is also in its proper position with respect to all other surveys in the system so that it can be used in the future for many other purposes. Of course, he should correct the co-ordinates for his survey by distributing his error of closure in latitudes and departures.

If he prefers to use azimuths instead of computed bearings, he first sets the vernier to read the given azimuth from the beginning monument to its azimuth mark and sights to that mark. Thereafter, instead of measuring the subsequent angles, he simply reads the azimuths of the various lines directly from the vernier plate and closes on the previously given azimuth at the terminating monument. Because the magnetic bearings will vary from corresponding plane ones (often considerably), in various parts of the State, the magnetic bearings of all lines, and the date of the survey, should be recorded.

The surveyor finds not only that the system can be used without difficulty but also that it simplifies the use of control data and gives a permanent general grid for the entire State. County boundaries, township boundaries, property boundaries, intersections of roads and streets, and any prominent features of a region can be located accurately with definite X and Y-co-ordinates. These plane co-ordinates can be readily transformed into latitude and longitude, and the point can thus be definitely located in the network of meridians and parallels that serve to locate points in the earth's surface. If, in some manner, the marker at such a point should be destroyed, it could be relocated definitely on the ground from its relation to other marked points. This is an important characteristic and one that should be given due consideration by all engineers and surveyors.

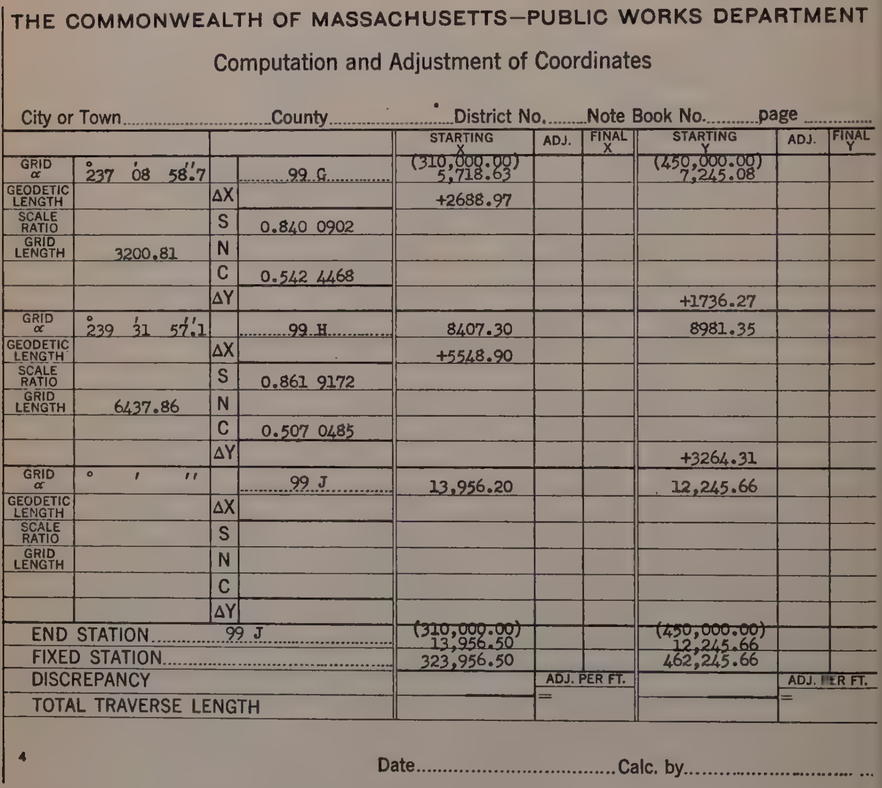
There have been strong objections to the use of grid co-ordinates based on the State system because of the large numbers involved. These large numbers were used in the State system only to avoid minus values in co-ordinates in any part of the State. In other words, the origin of the projection $X = 0$ and $Y = 0$ might be said to be an arbitrary point. Therefore, if the relation of the grid azimuth were maintained, one might use any set of X and Y-values.

The minimum requirements to compute an existing traverse, or tie it into the State system, are the grid co-ordinates of one marked station and grid azimuth from that marked station to another station visible from it. The co-ordinates are usually in hundred thousands, such as $X = 315\,718.63$ and $Y = 457\,245.08$, shown in Fig. 1. If the computer objects to carrying such large numbers in his computations, a value such as $X = 0$ and $Y = 0$ may be substituted for the known co-ordinate values of the marked starting station. In general practice, however, it is customary to assume some number sufficiently large so there will be no minus values such as 10 000 or 50 000. After computing the values of the traverse, the difference between the original co-ordinates and the starting co-ordinates should be added to the computed co-ordinates. This has been done in Fig. 1. The computations using natural lines and curves, designated by **S** and **C**, will be self-evident.

CO-ORDINATES BY SPECIAL TABLES

A table of plane co-ordinates of minute intersections of meridians and parallels of latitude was prepared to enable engineers to compute the co-ordinates of any geographic position by interpolating rather than using the longer direct method involving 10-place natural or logarithmic tables. While especially valuable for plotting purposes, the accuracy obtained in most cases will be sufficient for any survey, giving results checking within 0.01 ft.

The basic values for the projection in Massachusetts are the radius, in feet, from the origin of the cone of the projection, of parallels of latitude, and the value known as *Y*, in feet, of the length along the central meridian, of the intercepts between minute parallels of latitude. The value, in seconds, of θ, the angle at the origin of the cone between the central meridian and any other meridian, is also given.



THE COMMONWEALTH OF MASSACHUSETTS—PUBLIC WORKS DEPARTMENT

Computation and Adjustment of Coordinates

City or Town............................County.............................District No.........Note Book No............page ....

				STARTING X	ADJ.	FINAL X	STARTING Y	ADJ.	FINAL Y
GRID α	237° 08′ 58.7″		99 G	(310,000.00) 5,718.63			(450,000.00) 7,245.08		
GEODETIC LENGTH		ΔX		+2688.97					
SCALE RATIO		S	0.840 0902						
GRID LENGTH	3200.81	N							
		C	0.542 4468						
		ΔY					+1736.27		
GRID α	239° 31′ 57.1″		99 H	8407.30			8981.35		
GEODETIC LENGTH		ΔX		+5548.90					
SCALE RATIO		S	0.861 9172						
GRID LENGTH	6437.86	N							
		C	0.507 0485						
		ΔY					+3264.31		
GRID α	° ′ ″		99 J	13,956.20			12,245.66		
GEODETIC LENGTH		ΔX							
SCALE RATIO		S							
GRID LENGTH		N							
		C							
		ΔY							
END STATION........99 J				(310,000.00) 13,956.50			(450,000.00) 12,245.66		
FIXED STATION				323,956.50			462,245.66		
DISCREPANCY					ADJ. PER FT.			ADJ. PER FT.	
TOTAL TRAVERSE LENGTH					=			=	

4

Date..................................Calc. by..................................

FIG. 1

Using these basic values, the co-ordinate values of the intersection of every minute of latitude and longitude, within the State boundaries, were computed. This necessitated approximately 25 000 separate computations for the *X* and *Y*-co-ordinates of these minute intersections. These computations were somewhat simplified by the fact that they were started at the 71° 30′ meridian, known in the projection system as the prime meridian, which has an *X*-value of 600 000. Thus, for about 100′ east and west of the prime meridian, the computed distances from this point were added to or subtracted from 600 000—added when east of the prime meridian and subtracted when west.

Latitude	71° 03'				71° 04'				71° 05'			
° '	X $\Delta'x$	ΔX $\Delta_2 x$	Y $\Delta'y$	ΔY $\Delta_2 y$	X $\Delta'x$	ΔX $\Delta_2 x$	Y $\Delta'y$	ΔY $\Delta_2 y$	X $\Delta'x$	ΔX $\Delta_2 x$	Y $\Delta'y$	ΔY $\Delta_2 y$
41-45	722,798.61 -32.04	-4548.05 +1.18	273,620.06 +6,073.09	-23.55 0.00	718,250.56 -30.86	-4548.06 +1.19	273,596.51 +6,073.09	-22.66 0.00	713,702.50 -29.67	-4548.06 +1.19	273,573.85 +6,073.09	-21.78 +0.02
46	722,766.57 -32.04	-4546.87 +1.19	279,693.15 +6,073.08	-23.55 +0.01	718,219.70 -30.85	-4546.87 +1.19	279,669.60 +6,073.09	-22.66 +0.01	713,672.83 -29.66	-4546.87 +1.18	279,646.94 +6,073.10	-21.76 0.00
47	722,734.53 -32.04	-4545.68 +1.19	285,766.23 +6,073.10	-23.54 0.00	718,188.85 -30.85	-4545.68 +1.18	285,742.69 +6,073.10	-22.65 +0.01	713,643.17 -29.67	-4545.69 +1.19	285,720.04 +6,073.11	-21.76 0.00
48	722,702.49 -32.04	-4544.49 +1.18	291,839.33 +6,073.10	-23.54 +0.02	718,158.00 -30.86	-4544.50 +1.19	291,815.79 +6,073.12	-22.64 0.00	713,613.50 -29.67	-4544.50 +1.18	291,793.15 +6,073.12	-21.76 +0.01
49	722,670.45 -32.04	-4543.31 +1.19	297,912.43 +6,073.11	-23.52 0.00	718,127.14 -30.85	-4543.31 +1.18	297,888.91 +6,073.11	-22.64 0.00	713,583.83 -29.67	-4543.32 +1.19	297,866.27 +6,073.12	-21.75 0.00
41-50	722,638.41 -32.04	-4542.12 +1.19	303,985.54 +6,073.11	-23.52 +0.01	718,096.29 -30.85	-4542.13 +1.19	303,962.02 +6,073.12	-22.63 0.00	713,554.16 -29.66	-4542.13 +1.18	303,939.39 +6,073.12	-21.75 +0.01
51	722,606.37 -32.04	-4540.93 +1.18	310,058.65 +6,073.12	-23.51 0.00	718,065.44 -30.86	-4540.94 +1.19	310,035.14 +6,073.12	-22.63 +0.01	713,524.50 -29.67	-4540.95 +1.19	310,012.51 +6,073.13	-21.74 +0.01
52	722,574.33 -32.04	-4539.75 +1.19	316,131.77 +6,073.12	-23.51 +0.01	718,034.58 -30.85	-4539.75 +1.18	316,108.26 +6,073.13	-22.62 +0.01	713,494.83 -29.67	-4539.76 +1.19	316,085.64 +6,073.14	-21.73 0.00
53	722,542.29 -32.04	-4538.56 +1.18	322,204.89 +6,073.14	-23.50 0.00	718,003.73 -30.86	-4538.57 +1.20	322,181.39 +6,073.14	-22.61 +0.01	713,465.16 -29.66	-4538.57 +1.18	322,158.78 +6,073.15	-21.73 0.00
54	722,510.25 -32.04	-4537.38 +1.19	328,278.03 +6,073.14	-23.50 +0.02	717,972.87 -30.85	-4537.37 +1.18	328,254.53 +6,073.16	-22.60 -0.01	713,435.50 -29.67	-4537.39 +1.19	328,231.93 +6,073.15	-21.73 +0.02
41-55	722,478.21 -32.04	-4536.19 +1.19	334,351.17 +6,073.16	-23.48 0.00	717,942.02 -30.85	-4536.19 +1.18	334,327.69 +6,073.16	-22.61 +0.01	713,405.83 -29.67	-4536.20 +1.19	334,305.08 +6,073.17	-21.71 0.00
56	722,446.17 -32.04	-4535.00 +1.18	340,424.33 +6,073.17	-23.48 0.00	717,911.17 -30.86	-4535.01 +1.19	340,400.85 +6,073.17	-22.60 +0.01	713,376.16 -29.67	-4535.01 +1.19	340,378.25 +6,073.18	-21.71 0.00
57	722,414.13 -32.04	-4533.82 +1.19	346,497.50 +6,073.17	-23.48 +0.01	717,880.31 -30.85	-4533.82 +1.19	346,474.02 +6,073.18	-22.59 +0.01	713,346.49 -29.66	-4533.82 +1.18	346,451.43 +6,073.19	-21.71 +0.01
58	722,382.09 -32.05	-4532.63 +1.19	352,570.67 +6,073.19	-23.47 0.00	717,849.46 -30.86	-4532.63 +1.19	352,547.20 +6,073.19	-22.58 0.00	713,316.83 -29.67	-4532.64 +1.19	352,524.62 +6,073.19	-21.70 +0.01
59	722,350.04 -32.04	-4531.44 +1.19	358,643.86 +6,073.19	-23.47 +0.01	717,818.60 -30.85	-4531.44 +1.18	358,620.39 +6,073.20	-22.58 +0.01	713,287.16 -29.67	-4531.45 +1.19	358,597.81 +6,073.21	-21.69 0.00
42-00	722,318.00	-4530.25	364,717.05	-23.46	717,787.75	-4530.26	364,693.59	-22.57	713,257.49	-4530.26	364,671.02	-21.69

FIG. 2

When all the co-ordinate values for the minute intersections were computed, they were arranged in a tabular form of which Fig. 2 is a sample. A range of 15′ of latitude forms the side argument and 3′ of longitude forms the top argument. Each vertical column of 1′ of longitude is divided into four sections.

Section 1 gives the value of the co-ordinate, X, and between adjacent values, their increments, $\Delta'X$. Section 2 lists the difference, ΔX, in the value of X between each minute of longitude progressing easterly along a given parallel; and between these ΔX-values the differences, $\Delta_2 X$, of ΔX between successive minutes of latitude along a given meridian progressing northerly. Sections 3 and 4 are identical with Sections 1 and 2, except that the values listed are for

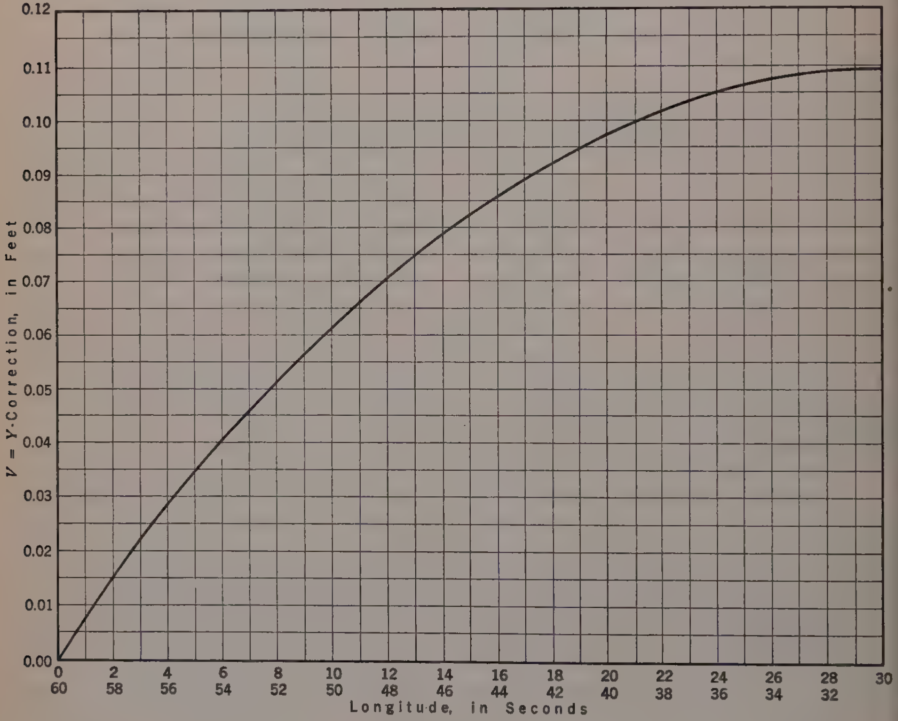


FIG. 3

the Y-co-ordinates. Thus, at a given latitude and longitude straight line differences can be found to the next larger minute of latitude and longitude. The complete table thus formed for the State of Massachusetts consisted of approximately 270 pages similar to Fig. 2, giving X and Y-values and their increments.

To convert to grid co-ordinates a given latitude and longitude listed to hundredths or thousandths of a second, it is only necessary to locate the required page in the table similar to Fig. 2, by use of a tabulated key, which gives the X and Y-values of the intersection of the given latitude and longitude, rounded off to the next lower full minute. Then, by multiplying the seconds, divided by 60, into the Δ-values and their increments found in the tables, a straight line interpolation can be made.

The *Y*-co-ordinates, as determined from the tables, are slightly in error because, between minute intersections, the parallels of latitude are assumed to be straight lines and actually they are curved. A graph was constructed (Fig. 3) giving corrections to compensate for this discrepancy. The values of *X* and *Y* as determined from the tables and the *Y*-correction are within 0.01 ft of those obtained through the use of the regular computation, using logarithms for changing geodetic positions to grid co-ordinates. Also, by the use of these tables, the time required to compute a grid co-ordinate is shortened by 65 to 75 per cent.

SAMPLE COMPUTATIONS:

Station: SOMMINT

Latitude 41° 46' 33".877
Longitude 71° 03' 15".650

$\phi = \frac{33.877}{60} = 0.5646167$

$\lambda = \frac{15.650}{60} = 0.2608333$

At longitude 71° 03' and latitude 41° 46'

$X_t = 722{,}766.57$; $\Delta' x_t = -32.04$; $\Delta X_t = -4546.87$; $\Delta_2 x_t = +1.19$
$Y_t = 279{,}693.15$; $\Delta' y_t = +6073.08$; $\Delta Y_t = -23.55$; $\Delta_2 y_t = +0.01$

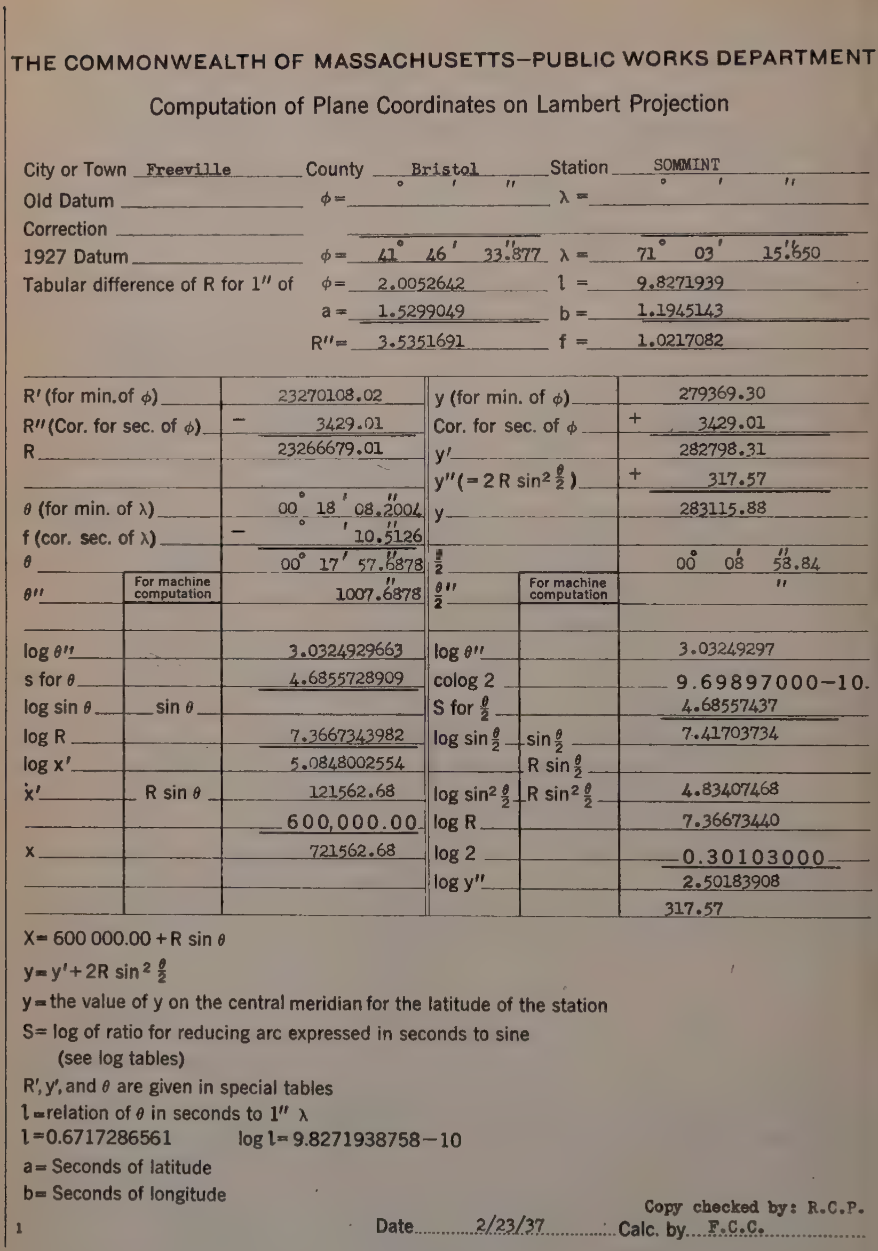
X:	722,766.57	
	-18.090	-32.04 x 0.5646167
	722,748.480	
	-1,185.800	0.2608333[-4546.87 + (+1.19 x 0.5646167)]
	721,562.680	
Y:	279,693.15	
	+3,428.962	+6073.08 x 0.5646167
	283,122.112	
	-6.141	0.2608333[-23.55 + (+0.01 x 0.5646167)]
	283,115.971	
	- .084	correction from graph
	283,115.887	

Adopted:
X = 721,562.68
Y = 283,115.89

By Computation:
X = 721,562.68
Y = 283,115.88

FIG. 4

If the intersection tables are used to obtain the co-ordinates when a geodetic position is given for the starting station, the other computations in a traverse need only 7-place logarithms or 7-place natural function tables, which is the same as required if the traverse were based on local co-ordinates. The only difference in computing traverse on the State system and on a local system is the starting azimuth. On the State system a simple computation, called a



THE COMMONWEALTH OF MASSACHUSETTS—PUBLIC WORKS DEPARTMENT

Computation of Plane Coordinates on Lambert Projection

City or Town Freeville County Bristol Station SOMMINT

Old Datum ______ ϕ = ° ′ ″ λ = ° ′ ″

Correction ______

1927 Datum ______ ϕ = 41° 46′ 33.877″ λ = 71° 03′ 15.650″

Tabular difference of R for 1″ of ϕ = 2.0052642 l = 9.8271939

a = 1.5299049 b = 1.1945143

R″ = 3.5351691 f = 1.0217082

R′ (for min. of ϕ)		23270108.02	y (for min. of ϕ)		279369.30
R″ (Cor. for sec. of ϕ)		− 3429.01	Cor. for sec. of ϕ		+ 3429.01
R		23266679.01	y′		282798.31
			y″ (= $2R\sin^2\frac{\theta}{2}$)		+ 317.57
θ (for min. of λ)		00° 18′ 08.2004″	y		283115.88
f (cor. sec. of λ)		− ° ′ 10.5126″			
θ		00° 17′ 57.6878″	$\frac{\theta}{2}$		00° 08′ 58.84″
θ''	For machine computation	1007.6878″	$\frac{\theta''}{2}$	For machine computation	″
log θ''		3.0324929663	log θ''		3.03249297
s for θ		4.6855728909	colog 2		9.69897000−10.
log sin θ	sin θ		S for $\frac{\theta}{2}$		4.68557437
log R		7.3667343982	log sin $\frac{\theta}{2}$	sin $\frac{\theta}{2}$	7.41703734
log x′		5.0848002554		R sin $\frac{\theta}{2}$	
x′	R sin θ	121562.68	log sin² $\frac{\theta}{2}$	R sin² $\frac{\theta}{2}$	4.83407468
		600,000.00	log R		7.36673440
x		721562.68	log 2		0.30103000
			log y″		2.50183908
					317.57

$X = 600\,000.00 + R\sin\theta$

$y = y' + 2R\sin^2\frac{\theta}{2}$

y = the value of y on the central meridian for the latitude of the station

S = log of ratio for reducing arc expressed in seconds to sine
(see log tables)

R′, y′, and θ are given in special tables

l = relation of θ in seconds to 1″ λ

l = 0.6717286561 log l = 9.8271938758 − 10

a = Seconds of latitude

b = Seconds of longitude

Copy checked by: R.C.P.

1 Date 2/23/37 Calc. by F.C.C.

FIG. 5

grid inverse, will give the azimuth and distance between two known stations. Using this inverse azimuth the work may proceed as with any other traverse.

A sample computation using the table of minute intersections (Fig. 2) and the graphic correction (Fig. 3) is given in Fig. 4. The chosen station, Sommint, is located so as to be covered by the sample page illustrated in Fig. 2. It is believed that the numerical work will be self-evident.

To check the results in Fig. 4, a computation of the co-ordinates for the same station by solution of the formula is given in Fig. 5, on the standard form of the Massachusetts survey. The detailed procedure needs no special comment. In particular, the close check with Fig. 3 is to be noted.

CONCLUSION

The value to all future land surveys of the rectangular system of plane co-ordinates will be in proportion to the local and community use made of Massachusetts Geodetic Survey work already completed and to be accomplished. Every effort is being advanced to provide each community in the State with fixed points of horizontal control together with the elevations above mean sea level.

Such stations are being located on individual town maps prepared for ready reference. Suitable symbols to identify the points are listed and explained in an accompanying legend. At each of these horizontal control stations the following data will be available: (1) A description of the general location; (2) a diagram showing distances and directions to station reference points; (3) plane co-ordinates on the State system; (4) direction angles, distances, and azimuths to adjacent stations; and (5) elevation above mean sea level.

Faithful use of this information will eliminate for all time multiplicity of origins, the over-supply of level datum planes, and the varied waste of land surveys, and will contribute to an enlarged network of fixed points whose permanence of location will rival that of the sun. It remains for the towns and small communities themselves to effect this realization. It remains for them to see to it that future public or private surveys are tied to and developed upon the data available at control stations; that the positions of all important points and of monuments are made permanently recoverable; that old established points are co-ordinated with the new control points and proper recomputations made; and that all future local work is tied to the State system. It likewise remains for these communities to appropriate sufficient funds to carry on this local work and to maintain complete records of procedure and results for future use.

Such is the story of the establishment of the system of rectangular co-ordinates in the State of Massachusetts and such is the path that must be followed for its complete extension and realization.

THE LAND COURT AND ITS ENGINEERING PROCEDURE

BY CLARENCE B. HUMPHREY,[3] ESQ.

SYNOPSIS

The purpose of this paper is to place before the engineer the work done by the Land Court; to acquaint him with its practice and procedure; to provide a background for his examination of title deeds; and to encourage the use of the Massachusetts Geodetic Survey triangulation points and traverse stations in connection with the State-wide co-ordinate system.

It is hoped that engineers connected with State and town governmental agencies in Massachusetts, knowing the present situation in regard to independent surveys, will co-operate and place their survey projects on this system, thus making available valuable map data.

From its beginning the business of the Court has increased at a uniform rate, falling off slightly during the depression period. The number of registration petitions filed is about 18 000 and certificates issued about 150 000. The decree and subdivision plans on file number about 25 000 and the assessed value of land registered is about $200 000 000.

EARLY GRANTS

The original source of title to the soil in New England, as in other parts of the United States, was in the English Crown which claimed it by the right of discovery and possession. The King was the owner and thus all titles to landed property were derived from a grant of the English Crown.

First, there the Royal Grants, among which might be mentioned the Plymouth Company in 1606; the Massachusetts Bay Company in 1629; the Ferdinando Gorges grant in 1639, covering most of Maine; the River Towns, in Connecticut, in 1662; the Province of New Hampshire in 1679; and the Narragansett Bay settlement under Roger Williams in 1643. In 1691 the Massachusetts Bay Colony received a new charter, and its jurisdiction covered what is now Maine and Massachusetts. The Plymouth settlement was included in the Massachusetts Bay Province thus reconstructed.

Next came the Colonial Grants with the power vested in the General Court. These grants were to individuals and groups of individuals who became known as proprietors. The grants to individuals were not numerous and seldom contained more than 350 acres, while the grants to the proprietors for laying out of plantations would contain a tract about 6 miles square. These plantation lands were surveyed and divided; and at legally called proprietors' meetings, selections of the divided parcels were made by drawing lots. Although these deeds were properly recorded the title itself runs direct from the proprietors' records.

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[3] Engr., Mass. Land Court, Boston, Mass.

RECORDING

Under the Massachusetts recording system [4] in vogue since 1634, the document recorded is a deed, which is operative without record, the title passing upon the delivery of the deed and before it is recorded.

According to the General Laws:[5]

> "A conveyance of an estate in fee simple, fee tail or for life, or a lease for more than seven years from the making thereof, shall not be valid as against any person, except the grantor or lessor, his heirs and devisees and persons having actual notice of it, unless it is recorded in the Registry of Deeds for the county or district in which the land to which it relates lies."

This Section although revised many times still reads very close to the original ordinance of 1640. The next paragraph, Section 5,[5] provides that:

> "The record of a deed, lease, power of attorney or other instrument, duly acknowledged or proved as provided in this chapter, and purporting to affect the title to land, shall be conclusive evidence of the delivery of such instrument, in favor of purchases for value without notice claiming thereunder."

These two Sections (Sections 4 and 5) are the very basis of the recording system, which not only preserves the evidence but gives a legal priority to the grantee of the recorded deed, insures a safe depository by the county, and makes it possible to investigate the title to property. First used in the Plymouth Colony in 1634, it remained in vogue until 1898 when it was modified by the adoption of the Land Registration Act.[6] The system proved well suited to the needs of the people and in a greater part of the Commonwealth it still would be satisfactory. However, with the passing of time, as cities became congested, as the records became exceedingly complicated, and as the methods of dealing with land have changed, real estate operators found it advisable to have their titles in such shape that they could be transferred speedily, the same as shares of stock, or other securities. For such purposes the existing system was not adequate. In the cities there was too much record title and in the country there was, at times, too little. Numerous other evils of the system needed correction.

Title to land is acquired in various ways, such as by inheritance, by will, by mortgage, and by direct purchase. During these changes over a period of years many things might occur to invalidate a title. Thus, no prudent man will purchase land until he has had the title studied by a competent examiner, as title on which money is loaned, or on which the issue of bonds is based, must be valid. If the land has repeatedly changed hands, it will be found that the title has been examined and re-examined by many separate interests with the resulting duplication of expense; and even then, the most exacting scrutiny might fail to disclose vital defects, such as prescriptive rights.

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[4] "The Origin of the System of Recording Deeds in America," by Joseph H. Beale, Jr. Harvard Univ., 1907.

[5] General Laws, Tercentenary Edition, Chapter 183, Section 4.

[6] "Land Registration Act of Massachusetts," by Charles Thornton Davis, Judge of Land Court. (Read before the New York State Bar Assoc., 1908.)

It was said by the Hon. Fred E. Crawford,[7] of the Boston Bar, that "if a new beginning could be made" back of which it would be unnecessary to go—and if a title could be given new life and started on a new career—much would be accomplished. In theory, all titles come from the State and if a title might be considered as coming anew from the State, then from that day the State would stand behind the title. This is what the Land Court does.

To Australia—the home of that other great principle, the Australian ballot—goes the credit for breaking the precedent in passing title to land. Sir Richard Torrens was at one time Commissioner of Customs in South Australia. Apparently annoyed at the delay in transfer of titles in land, due to the time consumed in the examination and re-examination of back titles, he devised a system of registration of title to land incorporated in what became known as "The Torrens Act."[8] This furnished the idea for legislation adopted in various parts of the United States.

The first American State to adopt some system of conveyancing by the registration of titles, was Illinois in 1895; then came Ohio in 1896; California in 1897; and Massachusetts in 1898. Since then Acts have been adopted in some form or other by Colorado, Minnesota, New York, North Carolina, Oregon, Washington, Utah, Virginia, North Dakota, South Dakota, and Georgia; also Hawaii and the Philippine Islands. The Act adopted by Massachusetts was distinctly local—drawn to fit existing conditions and establishing a system designed to give relief from the defects of the old method.

THE LAND COURT

In the Commonwealth of Massachusetts there are fourteen separate and distinct counties and twenty-one Registries of Deeds. The Massachusetts Act (now the General Laws, Tercentenary Edition, Chapter 185)[5] provides for a separate body, called the Land Court,[9] having State-wide jurisdiction, with a judge, two associate judges, and a recorder. This new system was grafted on to the existing recording system by creating a Land Registration Section in each Registry of Deeds, and making each Register an Assistant Recorder of the Court, *ex officio*.

The Court is situated in Boston, Mass., and its decrees are sent out for recording to the Assistant Recorders. On the authority of these decrees certificates are issued and all subsequent transfers are made by certificate. Thus, the title is given a new lease of life and a day is set, after which an examination is unnecessary. The Act not only provides machinery whereby ownership of the premises is adjudicated, but boundary lines on the earth's surface are judicially determined, and remain definite and fixed. To register a title the procedure is as follows:

(1) The petitioner files a petition and a plan with the recorder and pays the statutory fee. The petition describes the land as claimed; states all known encumbrances and adverse claimants with their addresses, whether admitted or

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[7] "The Massachusetts Land Court," by Fred E. Crawford, 1914.

[8] "The Torrens System," by Arnold Guyot Cameron, Houghton, Mifflin Co., Boston, Mass., 1915.

[9] "The Land Court: Being a Copy of the Land Registration Act, Chapter 185, General Laws (Tercentenary Edition) with Procedure and Practise," 1936.

denied; and gives the names and addresses of all adjoining owners. It is accompanied by an assessor's certificate signed by the local assessor verifying the adjoining owners as they appeared at the last assessment for municipal taxation. The plan is made from an actual ground survey and shows the land claimed with the location of the boundary lines. The petition is given a number, the first one filed being No. 1, and these numbers run consecutively throughout the State. All papers with the petition, as well as the plan itself, bear this same case number. At the same time, a notice of the action is filed in the registry of deeds.

(2) The petition is referred to a Court examiner to report on the state of the title; he is allowed two weeks for this work. He must be an attorney-at-law, appointed by the Court. When his report is filed, notice is given to all parties in interest by publication once each week for three successive weeks in a newspaper in the county where the land lies. It is expressly provided that by the description in the notice "to all whom it may concern," all the world are made parties defendant and are to be included by default, and that the registration proceeding is *in rem suam*. The return day of the notice must not be less than twenty nor more than sixty days from the date of issue, and within seven days the recorder sends an attested copy of the notice to each person named therein, by registered mail. An attested copy of the notice is also posted on the land itself by a deputy sheriff, not less than fourteen days before the return day. Such other and further notices are given as the Court may order.

(3) If on or before the return day no one appears to object to the registration of the title to the land in the name of the petitioner, the examiner's opinion with the complete abstract of the record title is carefully read and considered by the Judges. Even if no objections have been made, further service, or additional information, or both, are many times required. After all these precautions have been taken, an order for decree of confirmation and registration of the title is entered in the name of the petitioner.

(4) If on the return day appearances and answers have been filed objecting in part or in whole to the petitioner's claim for registration, then, on motion, the matters involved are set down for a formal hearing before the Court. If the objections are not disposed of in Court—by agreement, by stipulation, or by withdrawals—the case is heard and a decision filed by the Judge. This decision is final upon all questions of fact, but any questions of law arising in the Land Court may be taken directly to the Supreme Judicial Court within twenty days after the decision is filed. When remanded to the Land Court an order for decree is entered as in an uncontested case.

COST OF REGISTRATION

Upon filing a petition, the petitioner is required to deposit $25 plus one-tenth of 1% ($1 per $1 000) of the last assessment for taxation; and when the decree is issued he pays the further sum of $2 for the decree and copy, and one-fourth of 1% of the assessed valuation, but not less than $10, nor more than $1 000. At the same time, the recorder's office collects $5, which is transmitted to the registry with the decree, to pay for entering the papers and for preparing the certificate and owner's duplicate. In addition to these statutory charges, the petitioner pays his title examiner and also his engineer for survey work and plan. The $25 is for publication, sheriff's fees, etc., and if any surplus remains when the case is closed, it is returned to the petitioner. If such deposit is not sufficient, then the remainder is paid upon the request of the recorder and before any decree issues.

The one-tenth of 1% is paid as an assurance fund to be deposited with the State Treasurer who keeps it invested and adds the income to the principal; and when the fund amounts to $200 000, as it did in 1929, the income is used to help defray the expense of the Court. This fund is for the purpose of reimbursing the Commonwealth for any sum its Treasurer may be called upon to pay out because of negligence on the part of any officer under the Land Registration Act. The man given a title under a decree of the Court obtains an indefeasible title, one that is settled and that remains settled. This is most important.

SURVEYS AND PLAN WORK

The Land Registration Act does not provide for an engineer, but Judge Charles Thornton Davis, who has been with the Court from 1898 to 1936, early recognized the necessity for an engineering department. He established it as an adjunct to the Recorder's Department.[6]

Instructions as to surveys were issued to acquaint the engineers with the requirements of the Court in regard to surveys and plans, and to standardize Land Court surveys. This was necessary in order to obtain in the first instance full engineering data, showing field monuments, which would enable any engineer to reproduce the lines upon the ground. In the past the fault has been that plans filed in the local registries lacked both monuments and survey data and, at times, even distances on the property lines themselves. Plans showing property and subdivisions are still recorded which have little or no value because the lines shown on them are in dispute and the engineer who made them has long since been dead. It is a case in which the engineer considered the missing data as a part of his stock in trade. This condition, however, should not discourage the filing of such plans, because some deeds are so ambiguous that even a sketch of the land sought to be described would be of assistance.

What is a survey? What are the requirements for a model plan and what data should be shown thereon? To be of any value, a plan should show plotted lines drawn to scale, distances, angles or bearings, survey notes when necessary, and the character of existing physical boundaries and monuments. Thus, all lines may be reproduced accurately by another engineer many years hence should occasion require.

In making plans for the Land Court:

(1) The plan must be made on tracing cloth.

(2) The sheet may be 18 in. by 24 in., or 24 in. by 36 in., or any convenient larger size.

(3) The title must state the location, scale, date, name, and address of the surveyor.

(4) The plan must show the north point, town, or county lines, names of adjoining owners, passageways, street lines, boundary monuments, all fences,

walls, buildings on or adjacent to lines claimed (using proper conventional signs), and all other natural monuments pertinent to the purpose.

(5) The scale must be such as to show clearly all data necessary.

(6) The area may be by calculation or by scale; if by scale the sign, ±, is affixed.

(7) Such parts of buildings and other physical features on adjoining land as are within 10 ft of boundary lines are shown.

(8) Boundaries of the property, pertinent dimensions, and all physical features are shown in black; all traverse and surveying data in red.

(9) The plan must indicate plainly how each corner, property line, and reference point is marked on the ground. This information is essential, for the reason that respondents often object to a plan and survey because a stone bound, partition wall, retaining wall, or other object used to mark a boundary, has been erroneously included or excluded. On this account careful inquiry should be made to determine the ownership of such boundary monuments. If necessary a note is included describing what part of the boundary monument is used.

(10) The parcel sought to be registered must be enclosed within blue tinting.

(11) The plan should be oriented so that the top is in a general northerly direction.

Engineers are always reminded that the land may be bounded on ways, that is, streets, avenues, highways, boulevards, and parkways, also on railroad locations and public lands. Such ways are monuments, possibly referred to in the title deeds and as such the data are, or should be, deposited in the various public offices and the county registries. They are asked to examine these data and to place on the plan the initial data points used in making the survey.

In regard to bearings, engineers are asked to carry forward, if possible, one bearing used in a previous registration case; if not, to use a bearing found on an official street plan; or, as a last resort, to read the magnetic bearing of some one line of the new survey. All bearings on the survey must be relative, computed from measured angles.

A computation sheet is required, showing traverse on the lines claimed, or on lines from which the lines claimed are located. It gives computed latitude and departures and additional data, if necessary and not shown upon the plan, so that rectangular co-ordinates of all points on the plan can be computed.

On the plan is the endorsement: "I certify that this survey was made in accordance with Land Court instructions," with the surveyor's signature. Thus, the engineer must have all these requirements in mind when making his survey.

This plan is indexed and casually examined; if any data are missing or monuments should be set, the attorney of record is so notified. This procedure expedites matters when the case is ready for decree, which may be months later depending on the questions involved. The plan is used throughout the hearings. Usually, no changes are made; in some cases the lines are changed by agreements; and in still other cases, due to protracted hearings and formal decisions, many changes are made. When, however, an order for final decree is issued a plan called the decree plan is prepared in the Engineering Department. These plans are all of one uniform size, 10 in. by 15½ in.

This decree plan is sufficient in itself, with all survey data and monuments shown on it, to enable the established lines to be found on the ground. From this plan the decree is drafted describing the land as registered and setting forth all encumbrances, easements, or appurtenant rights, as set out in the order for decree. The Court early decided to use a bounding description rather than a so-called running description and, by the plan reference, to make the plan itself a part of the decree of the Court. The Supreme Court has often held that, by a well settled rule of construction where a plan is referred to in a deed, the lines, courses, distances, references to monuments, and other particulars appearing on the plan, are to be regarded as much as they would be if expressly recited in the deed.

The descriptions in the decrees, therefore, are brief, using a general bounding description, grouping together shorter lines running in the same general direction, but reciting, however, the adjacent owners and natural boundaries. The first boundary is usually a highway; thence the description reads clockwise. The plan reference most often used is as follows:

> "All of said boundaries are determined by the Court to be located as shown upon plan numbered * * *, which will be filed with the original Certificate of Title issued on this decree, the same being compiled from a plan drawn by John Doe, Engineer, dated * * *, and additional data on file in the Land Registration Office, all as modified and approved by the Court."

The decree and plan are sent to the assistant recorder for the district within which the land is situated. He transcribes this decree in a registration book and this entry is the original certificate which is accompanied by the plan. The assistant recorder then makes an exact copy of the original certificate, adding to it the words, "Owner's Duplicate Certificate," and delivers this to the owner.

Thus, the title to the parcel of land may be considered as being born again. From this time it is guaranteed by the State. The parcel thus started on a new career may contain only a few hundred square feet, or it may comprise several square miles, as the case may be.

GEODETIC CONTROL

From the beginning in Massachusetts it has been the aim to have all Land Court surveys made so that co-ordinate values could be computed relative to an assumed origin which might be one corner of the locus. The early instructions asked to have all surveys connected with recognized triangulation points when within or near the locus. When not connected with any recognized origin, then an origin within the survey is assumed and contiguous surveys is connected with it. This is carried on until the group of surveys, in turn, is connected with some local origin connected with the triangulation system, and then when these lands are thereafter dealt with, the lines are recomputed, based on the local origin. No request has been made to have local engineers use geodetic computations. Very few are acquainted with the formulas and they do not associate values in degrees, minutes, and seconds with small surveys, but rather with the U. S. Coast and Geodetic Survey work. If they are to carry on this horizontal system of control it must be made easy for them. Thus, the calculations for all reductions from geodetic values to plane co-ordinates have been simplified by the methods of the Massachusetts Geodetic Survey, as explained in Mr. Houdlette's paper. In many cases the necessary co-ordinates are furnished them before they begin a survey, especially if it covers a large area.

With the State divided into twenty-one registry districts, the system first adopted was to have an origin in the center of each so that all values would be relative to land registered and recorded in the respective registry of deeds. This system of rectangular co-ordinates was based upon a plane tangent to the earth's surface at a point. The area of each district was such that no appreciable error would appear on the exterior perimeter. The origin being a part of the State Triangulation system, the reduction from geodetic co-ordinates to plane co-ordinates was first made using the L.M.Z. formula (L.M.Z. has been adopted as representing latitude, longitude, and azimuth) and, later, by using formulas in *Special Publication No. 71*, issued by the United States Department of Commerce in 1921. This system was gradually being utilized by local engineers and by engineers working on the State highways.

In April, 1933, working in conjunction with the Department of Public Works a new system was adopted, as explained by Mr. Houdlette, whereby the State was divided into eight zones, but still using the plane rectangular system. The adoption of these new zones made it easier for the State engineers in crossing from one system to the other, as the number of origins were reduced from 21 to 8. It was also a step forward in having the highway network placed on a definite system referred to a common meridian, and if co-ordinate values were carried forward on subsequent highway plans they would have an economic value, appreciated by the engineer, but little realized by the owner of the land.

In December, the U. S. Coast and Geodetic Survey reported that all future values for triangulation stations in Massachusetts would be computed and published, as based on a new datum known as the North American datum, 1927. In using Government maps and geographical positions given in terms of latitude and longitude, especially in research work, engineers should, therefore, have in mind the three main projections used in Massachusetts, namely the Bessel spheroid used previous to 1880; the North American datum based on the Clarke spheroid adopted in 1880; and the North American datum, 1927.

In consequence, it was decided to adopt one uniform system of co-ordinates with values expressed in feet and fractions thereof, extending across the State. The projection advised by Dr. O. S. Adams, U. S. Coast and Geodetic Survey, as approved by the Massachusetts Department of Public Works and the Land Court, is defined as the Lambert Conformal Conic Projection of Clarke's spheroid of 1866, with two standard parallels held true to scale. This system is fully described by Mr. Houdlette.

CADASTRAL MAPS

The need for key maps upon which to plot the location of registered parcels was early recognized. All the available commercial atlases of the towns, cities, and counties were purchased and upon these atlas sheets the lands sought to be registered were outlined in blue and the case number indicated. These atlases serve as a graphical index, not only for the use of the Court, but for the public as well, to ascertain whether any registered lands exist in any given locality.

In addition to these atlases, there have been made from time to time standard base maps, called cadastral plans. The basis for these plans is the geodetic control system and the fifty-four Topographical Sheets of the U. S. Geological Survey, which include the Commonwealth of Massachusetts.

These 15′ quadrangle sheets have been divided into nine lettered sections of 5′ each and each section is further divided into eight numbered plans. Thus, there are seventy-two plans to a topographical sheet, and the area covered by each plan is 2′ 30″ of longitude and 1′ 15″ of latitude. The scale for these plans is 300 ft to an inch which allows the use of standard paper, 27 in. by 40 in., with about 1-in. margin. On a decree plan the reference, "52-H-6," would indicate that the land as registered is plotted on Topographical Sheet 52, Section H, Plan 6.

These plans when first prepared show border lines of longitude and latitude, dimensions, in feet, on each side, and the rectangular co-ordinates of each corner intersection, as well as the sheet number, town names, and town boundaries. From these data, inside construction lines are plotted and the sheet is then ready for plotting any parcel or lines connected with the geodetic control. Existing maps upon which the geodetic lines appear may be easily enlarged and placed upon these plans.

SUBDIVISION PLANS

The Land Registration Act provides that a registered owner holding one certificate for several distinct parcels may surrender it and take out a certificate for each parcel; likewise, if he holds separate certificates for each parcel, he may surrender them and take out one certificate for all parcels or any number of them. The Act further provides that an owner who subdivides a tract of registered land into two or more lots shall file with the Court a plan thereof, when applying for a new certificate; that this plan shall be verified; and that all newly created boundaries and ways shall be distinctly and accurately shown thereon.

The original Certificate of Title describes with a bounding description the perimeter of the land registered, and this is true whether the decree plan shows one parcel containing 5 000 sq ft of land or 5 acres subdivided into, say, thirty numbered parcels. In the first case, if the parcel was conveyed, the owner would surrender his certificate; it would be canceled and a new certificate issued to the new owner, referring to the plan filed with the first certificate. This might happen many times.

In the second case, one or all of the thirty separate parcels as shown upon the decree plan might be conveyed to as many separate new owners. Each new owner would receive a certificate describing the parcel he had acquired and referring to it by Parcel No. * * *, Plan No. * * *, filed with Certificate of Title No. * * *. As each parcel was conveyed, a memorandum to that effect would be entered on the grantor's certificate giving the grantee's name, date, and other pertinent information. Likewise, all mortgages or other papers affecting the title would be entered, so that the state of the owner's title is always obtainable by first reading the owner's certificate and thus observing the nature of the encumbrances entered on its reverse. In many cases where mortgages have been registered, later discharged, and the parcels sold, the owner will surrender his certificate encumbered with all these references and ask that he have a new certificate, purged of all "dead wood," which after an examination by the Court is granted.

When the decree plan shows numbered parcels, or parcels and ways, separate Certificates of Title may be issued without any further Court order. The fact that the divisions are on the decree plan is the Assistant Recorder's authority. When, however, any parcel as registered and shown upon the decree plan is changed by creating new interior boundary lines, a new plan is required as described, and, at this time, the owner files with the Engineering Department of the Court his Certificate of Title, and the new plan. This certificate is examined to see if it covers the land shown on the plan as filed, and the plan itself is verified to see that it is consistent with the previous registration plans to which the certificate refers and that all new boundaries and ways are accurately shown with proper engineering data, the same as if a new petitioner's plan were being filed for original registration. Each new lot or parcel must be consistent in itself and the distances and angles indicated must be reasonable. If found correct in all these details it is approved and a plan made from it on standard-size white paper showing the new divisions with data appearing on the filed plan, and in many cases additional data appearing on the files of the Court. On this plan appears the legend:

> "Separate Certificates of Title may be issued for the numbered lots as shown hereon; or it may be for 'certain lots, *viz.*, Lots 24 and 25 as shown hereon'; as the case may be. This legend is attested by the Court."

This plan with the owner's Certificate of Title is sent to the Assistant Recorder for the district in which the land is situated with an order to cancel that part of any plan or plans upon which the land thus divided is shown. This process continues *ad infinitum.*

The subdivision plan work is an important and ever-increasing function of the Court. It is handled exclusively in the Engineering Department where in many instances complete descriptions based upon the new plan are furnished to the Assistant Recorders. The subdividing of land, the relocation of highways, the merging of contiguous parcels into one ownership, the re-arrangement of lots, the call for plans showing land remaining after a large tract has been actively dealt with, and the many other activities encountered in dealing with land and title create new plan work and the issuance of new Certificates of Title. This cancellation of the old or previous plans and the making of new plans is the means of continually keeping up to date the existing state of the title as well as the existing facts upon the ground, with newly acquired engineering data.

It is in dealing with subdivision plan work and the calculations incident thereto that the importance of a rectangular co-ordinate system, whether local or not, is impressed on the engineer. In many instances, the adoption by the State of one common projection to be used by all engineering agencies will prevent the duplication of survey work, which all engineers know exists at the present time.

Although a Land Court will quiet the title and make it indefeasible, a coordinate system such as is now recommended by the U. S. Coast and Geodetic Survey furnishes the foundation for all local and municipal surveys; eliminates the uncertainty of the magnetic meridian; provides a system for placing on plans the exact value of all corners, thus making property plans more valuable; provides a method of reproducing lines in areas devastated by fires; and, finally, tends to reduce materially the boundary line controversies which are so detrimental to the peace of a community, as a single point on the earth's surface furnishes a starting point for all surveys. Most disputes arise from a misinterpretation of the law controlling boundaries and a difference of opinion between engineers as to the starting point.

Doubtless problems of this general type were in the mind of Moses when he stated in "Deuteronomy" 27:17: "Cursed be he that removeth his neighbour's landmark, and all the people shall say, Amen."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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REPORTS

LAND SURVEYS AND TITLES

FIRST PROGRESS REPORT OF THE JOINT COMMITTEE OF THE REAL PROPERTY DIVISION, AMERICAN BAR ASSOCIATION AND THE SURVEYING AND MAPPING DIVISION, AMERICAN SOCIETY OF CIVIL ENGINEERS

Introduction

Although a great portion of wealth is represented by land, little progress has been made in the United States toward workable methods of recording either the title or the location of land boundaries. The methods used to-day were developed when land was comparatively inexpensive and population densities were low. With the passage of time and the concentration of population in metropolitan areas, the necessary records concerning land title and land boundaries have multiplied with increasing rapidity. Title search for individual parcels is becoming more arduous and the difficulty and cost of compiling data necessary for the determination of boundary location preclude proper survey procedure, despite the development of modern precise surveying methods. Compiled data can seldom be reconciled with geometrical requirements imposed by the existing land, and the location of boundaries, therefore, is more often a matter of judgment than of skill.

The Committee has been organized to study these conditions and to make recommendations for improvement.

Recommendations

1.—At the beginning of the Committee investigation it was at once evident: (*a*) That the importance of the problems under consideration and the difficulties involved in the solution of them necessitate a considerable period of study; and (*b*) that in order to avoid errors, a considerable period of time will be necessary for proper criticism and revision of the final report.

It is recommended, therefore, that the Committee be continued for a period of at least three years, provided, however, that preliminary reports be submitted from time to time as the work progresses.

Note.—Adopted by the Committee of the Surveying and Mapping Division on Land Surveys and Titles, at its meeting in New York, N. Y., March 30, 1938. Written comments are invited for immediate publication in *Proceedings*.

2.—As the solution of the problem is dependent not only upon fact-finding but also upon the development of a plan for improvement, it is recommended that the Committee be relieved of the necessity of reporting the particular steps leading to that plan, to the end that Committee deliberations will not be mistaken for final recommendations.

3.—The studies of the Committee have shown that one of the basic causes of the discrepancies and inexactitudes usually present in boundary location is the lack of permanently fixed reference directions and positions—consistently inter-related, and permanently marked—which are publicly known and legally recognized. Without such fixed directions and positions, land boundaries are at best dependent upon individual monuments and landmarks subject to removal or loss, seldom publicly recognized or inter-connected by surveys so as to obviate geometric impossibilities or loss of the marks; and, therefore, nothing exists upon which descriptions can be based nor by which independent locations can be established on the ground. It is only by the acquiescence of interested parties that a property line can be said to be established permanently.

The Committee finds further that, in order to provide proper reference marks for fixing position and direction, a system of survey monuments should be established, connected by properly controlled precise surveys, and that the positions of these monuments, so determined, should be expressed in terms of a co-ordinate system, preferably by plane rectangular co-ordinates. Descriptions of the monuments, together with their co-ordinates, must be recorded by some public authority and the record made available for public use.

The United States Coast and Geodetic Survey has developed a plan for the establishment of such inter-related monuments and systems of plane co-ordinates for defining positions. This plan has already been put in operation in several States and has won the immediate approval of practicing land surveyors and title examiners. Many thousands of monuments have been set and their co-ordinates have been determined. The plan is known as the "State System of Plane Coordinates." It has been thoroughly studied by the members of the Committee, and it is believed to offer an adequate remedy for the deficiencies described. The utility of the plan extends to so many fields other than that of land surveys and descriptions that the cost to the taxpayer for its adoption will be returned manyfold. It provides a basis for checking and controlling instrumental and aerial surveys for maps and plans required for all engineering development. The time and cost of establishing special control for each project are thus obviated. Furthermore, all existing data can be compiled, and boundary lines can be correlated without field determination.

THE STATE SYSTEM OF PLANE CO-ORDINATES

The entire United States is covered by series of triangulation stations which are physical marks on the ground. Most of the stations consist of concrete shafts, 4 ft or 5 ft long, set nearly flush with the ground, with bronze disks set in their tops which identify them and also contain a small hole which represents the exact position. Buried under each shaft is a short cylinder of concrete with another disk set directly under the surface disk. Usually, three more shafts with disks are set near-by and witness measurements are recorded by which the station can be replaced precisely if destroyed. In rock outcrops the disks are grouted in holes in the rock.

All these triangulation stations are connected by precise surveys so that their relative positions are determined accurately. Thus, even if all the marks of many stations were removed, it would still be possible to replace them in their original positions.

Since this net of triangulation stations extends over so large an area—the entire United States—it is necessary, due to the curvature of the earth, to express the positions of the stations in terms of a spherical co-ordinate system; that is, by precise latitude and longitude. As it is unnecessary to consider the curvature of the earth in the reduction of most survey data, the mathematics of this procedure is unfamiliar to all but a few specialists. It is clear then, in order to make them generally usable, that some form of plane rectangular co-ordinates must also be used to express the position of these stations. The U. S. Coast and Geodetic Survey has developed a plane co-ordinate system for every State, according to which the positions of the triangulation stations may be expressed. In the larger States several zones have been introduced, each zone having a separate origin. The zones are made as large as possible without introducing appreciable differences, caused by earth curvature, between measurements on the ground and distances as expressed by the co-ordinates. The variations thus introduced are so slight that they cannot be discovered by ordinary survey methods.

It is planned that monuments will be established in convenient locations along highways and streets, and precise surveys will be made connecting these monuments with triangulation stations. The co-ordinates of the monuments can be determined by using the plane co-ordinates of the triangulation stations. In many States this work is well under way, and many municipalities are extending the monument system throughout their streets. By this plan the position of every monument in the system can be determined with relation to that of every other monument, so that it is inconceivable that the position of the system should ever be lost.

To utilize the system a land surveyor must make a proper survey connection with two monuments. He can then either state the plane co-ordinates of the property corners in his description, or give the bearings and distances of two or more property corners from the monuments. With these data in the description there can be no question as to the precise position of the property. Its location is described accurately and permanently. Moreover, it is possible to determine, by examination of descriptions of this kind, whether all the parcels lotted in a certain tract will, in fact, fit into the tract.

When once the title examiner has become familiar with such a system, he can determine, without a field survey and a surveyor's guess, whether the requisite land exists for the title he is examining. Surveys are thus co-ordinated, and, in fact, made part of one great survey which is all-inclusive.

There are many applications of the system to engineering and mapping. It has proved of extreme value for tax maps and has been used considerably for that purpose.

It is desirable that each State legislature enact enabling laws defining the system used in the State and naming it so that any co-ordinates used can be referred to by name. A model act has been prepared and an example of it is attached hereto as an Appendix. Certain obvious changes are necessary, of course, to adapt the act to other States.

It is also desirable that a bureau of surveying and mapping be established in each State to administer the co-ordinate system. Such a bureau should be the depository of control-survey data, should disseminate it to the public, and should maintain the monuments. The bureau should be equipped to aid counties, municipalities, and other political subdivisions, to extend the monumentation along the roads and streets. By proper supervision of the work and by careful checking of the results, the bureau would be able to include these surveys in the State systems.

The Committee recommends, therefore, that each State legislature enact an enabling law as described herein and designate by law the establishment of a bureau of surveys either independent of existing State Departments or as a division of an existing department. The necessary appropriations should be made at the same time to start the bureau, to be increased as the work of the bureau results in savings in survey costs and litigation.

Respectfully submitted,

DORR VIELE, *Chairman,*
Committee of the Real Property
Division, American Bar Association;

PHILIP KISSAM, *Chairman,*
A. H. HOLT, *Secretary,*
Committee on Land Surveys and
Titles, Surveying and Mapping
Division.

MARCH 30, 1938

APPENDIX

A BILL FOR

An Act to describe, define, officially name and regulate the use of a system of coordinates for designating and stating positions of points on the surface of the earth within the state of Iowa.

Whereas, the United States Coast and Geodetic Survey has established a system of plane coordinates, tied to the national triangulation net, for the purpose of making available for general public use a simple, convenient, state-wide system for stating the location of points on the earth's surface, such as monuments, section corners, property corners, highway markers or any other point with which a survey might be concerned; and

Whereas, It is desirable to assign officially a name to this system in order to so define the system as to make certain its meaning when the name is used; and

Whereas, It is desirable to regulate the filing of data expressed in terms of the system in order to insure reliability of such data when permitted to become a part of the public records; therefore

Be It Enacted by the General Assembly of the State of Iowa:

Section 1. The system of plane rectangular coordinates which has been established by the United States Coast and Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the state of Iowa is hereafter to be known and designated as the "Iowa Coordinate System."

For the purposes of the use of this system the state is divided into a "North Zone" and a "South Zone."

The area now included in the following counties shall constitute the North Zone. (Here are given the names of counties included in this zone.)

The area now included in the following counties shall constitute the South Zone. (Here are given the names of the counties included in this zone.)

Section 2. As established for use in the North Zone, the Iowa Coordinate System shall be named, and in any land description in which it is used it shall be designated, the "Iowa Coordinate System, North Zone."

As established for use in the South Zone, the Iowa Coordinate System shall be named, and in any land description in which it is used it shall be designated, the "Iowa Coordinate System, South Zone."

Section 3. The plane rectangular coordinates of a point on the earth's surface, to be used in expressing the position or location of such point in the appropriate zone of this system, shall consist of two distances, expressed in feet and decimals of a foot. One of these distances, to be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. These coordinates shall be made to depend upon and conform to the plane rectangular coordinates of the triangulation and traverse stations of the United States Coast and Geodetic Survey within the state of Iowa, as these coordinates have been determined by the said Survey.

Section 4. When any tract of land to be defined by a single description extends from either of the two coordinate zones into the other zone, the positions of all points on its boundaries may be referred to either of these two zones, the zone which is used being specifically named in the description.

Section 5. For the purpose of more precisely defining Iowa Coordinate System, the following definition by the United States Coast and Geodetic Survey is adopted:

The Iowa Coordinate System, North Zone, consists of a Lambert Conformal projection of the Clarke spheroid of 1866, having a central meridian 93° 30′ west of Greenwich. The intersecting cone of this projection cuts the surface of the spheroid in parallels of latitude 42° 04′ and 43° 16′ north of the equator, along which parallels the scale shall be exact. The origin of coordinates for this zone is at the intersection of the meridian 93° 30′ west longitude and the parallel 41° 31′ north latitude. This origin is given the coordinates: $x = 2{,}000{,}000$ feet; $y = 0$ feet.

The Iowa Coordinate System, South Zone, consists of a Lambert conformal projection of the Clarke spheroid of 1866, having a central meridian 93° 30′ west of Greenwich. The intersecting cone of this projection cuts the surface of the spheroid in parallels of latitude 40° 37′ and 41° 47′ north of the equator, along which parallels the scale shall be exact. The origin of coordinates for this zone is at the intersection of the meridian 93° 30′ west longitude and the parallel 40° 00′ north latitude. This origin is given the coordinates: $x = 2{,}000{,}000$ feet; $y = 0$ feet.

The position of the Iowa Coordinate System shall be as marked on the ground by triangulation or traverse stations established in conformity with the standards adopted by the United States Coast and Geodetic Survey for first-order and second-order work, whose geodetic positions have been rigidly adjusted on the North American datum of 1927, and whose plane coordinates have been computed on the system here defined. Any such station may be used for establishing a survey connection with the Iowa Coordinate System.

Section 6. No coordinates based on the Iowa Coordinate System, purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within one-half mile of a triangulation or traverse station established as prescribed in Section 5 of this act.

Section 7. The use of the term, "Iowa Coordinate System," on any map, report of survey, or other document, shall be limited to coordinates based on the Iowa Coordinate System as defined in this act.

Section 8. Nothing contained in this act shall be interpreted as requiring any purchaser or mortgagee to rely wholly on a description based on the Iowa Coordinate System.

Section 9. If any provision of this act shall be declared invalid, such invalidity shall not affect the validity of any portion of this act which can be given effect without such invalid part.

Section 10. This act, being deemed of immediate importance, shall be in full force and effect from and after its publication in the *Tipton Conservative*, a newspaper published at Tipton, Iowa, and the *Iowa City Press-Citizen*, a newspaper published at Iowa City, Iowa.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY J. D. GALLOWAY, M. AM. SOC. C. E.

J. D. GALLOWAY,[47] M. AM. SOC. C. E. (by letter).[47a]—It is gratifying to note that a number of engineers have found time to discuss this paper and to contribute valuable information. It was to be expected that differences of opinion would develop, but it is to be noted that the discussion refers mostly to details rather than to the basic elements of design.

The statement that rock-fill dams should consist of a loose fill with slopes on both faces closely approximating natural slopes, with an impervious facing on the up-stream side between which and the rock-fill there should be placed a cushion of dry rubble, was not questioned. In this feature it may be assumed that there is unanimity of opinion. No one advocated placing a core-wall in the center of the dam, as experience has shown that the center of the cross-section is not the place for the impervious element.

There was some discussion of the definition of a rock-fill dam. The writer purposely restricted the definition as given, although that is a matter of opinion. Dams have been built with an earth section, supported on the down-stream side by a rock-fill. It would seem that such a dam is really an earth-fill. The resistance to the water is taken by the earth-fill in accordance with the design of that type of dam, and the rock-fill below is little more than earth of larger grains. It is sufficient for the purposes of the paper to limit the discussion to dams as described.

Mr. Pearce gives a good reason for making the up-stream slope approximately the natural slope of the rock-fill. Steeper slopes have been used, but in all cases, it was necessary to retain the loose rock in place by carrying up the

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NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston; January, 1938, by Harold K. Fox, M. Am. Soc. C. E.; February, 1938, by Messrs. Charles H. Paul, and A. Floris; March, 1938, by Messrs. Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, and L. F. Harza; April, 1938, by Messrs. Paul Baumann, O. W. Peterson, and George W. Howson; May, 1938, by Messrs. John E. Field, John H. Wilson, Frederick H. Fowler, I. C. Steele and Walter Dreyer, and F. Knapp; June, 1938, by Messrs. Ralph J. Reed, F. J. Sanger, and C. S. Jarvis; and September, 1938, by Messrs. L. R. East, and Francisco Gomez-Perez and Miguel Jinich.

[47] Cons. Engr., San Francisco, Calif.

[47a] Received by the Secretary October 3, 1938.

dry rubble at the same time. This was done in the earlier dams, but the upstream slopes of recent dams have rendered this procedure largely unnecessary. The suggestion was made that the horizontal joints might not be in compression, due to the natural shrinkage of the concrete in setting. This condition might obtain for a time, but it is believed that the settlement of the rock will proceed at a rate greater than the concrete will contract, thus placing the concrete in compression at the horizontal joints.

Among other points mentioned by Mr. Muckleston, he suggests the possibility of using puddled clay loaded with rock as the impervious element. It is the opinion of the writer that nothing of the nature of clay should ever be used in a rock-fill dam. When wet, as it would be, the clay will moisten and will slide from its position. Any rock loaded on top of the clay will be squeezed into it and will cause undue settlement. The same objection may be made to any form of asphalt concrete for the impervious element, as suggested by Mr. Muckleston. Although it is flexible, it will tend to flow under pressure and, in time, will lose its life from contact with water or under the influence of the sun's heat.

Mr. Paul refers to the English Dam and asks how it happened to be destroyed. Of this, the writer has no information except that given by the late A. J. Bowie,[48] M. Am. Soc. C. E. The general opinion was that the initial failure of the dam was caused by dynamite, as there was an active contest by different mining companies for water on the Yuba River at that time. However, the failure may have been due to the decay of the log cribs within the structure.

Mr. Floris calls attention to the fact that on an inclined face the weight of water adds to the stability of the dam. It was the writer's intention to convey the same idea, a fact that is common to all dams where the impervious element is next to the water. It was so expressed in the paper (see heading, "Design: Cross-Section") where the sliding factors are given and the water load included as a vertical force that acts by increasing friction to resist the horizontal component tending to slide the dam. In stating that rock-fill dams resist water pressure by their weight only, it was intended to mean that anything of the nature of arch or cantilever action, such as may be present in masonry dams, is not possible.

Dean Morris, in his interesting discussion of the paper, draws attention to the possible effect of earthquakes upon a rock-fill dam and suggests the possibility of large and unequal settlements taking place when the dam is shaken by the waves. Earthquake effects may be of two different kinds. Practically all earthquakes are tectonic; that is, they are caused by an actual rupture of the earth's crust, that may or may not extend to the surface of the ground. Differential movements along the earthquake rift may be large. In the California earthquake of 1906 the maximum differential movement along the San Andreas fault was about 22 ft and the maximum vertical movement, about 3 ft. It is obvious that a dam of the type under discussion would be badly broken if situated on the rift, especially if built with any form of facing mentioned in the paper, providing the movement took place under the dam. It is more than probable that the dam would be destroyed. Earthquake rifts, as a rule, are not single cracks in the earth, but there is usually a zone that may be of a width up to half a mile or so, where, in the past, the earth has been subject to crushing and movement. On account of the crushing of the rock, earthquake rifts or valleys are usually the site of streams where reservoirs are required for storage. The Coast Mountains of California, including the San Gabriel Range, are of this type where numerous earthquake rifts are found. It is the writer's firm opinion that rock-fill dams should never be built in such places. The type does not lend itself to the inclusion of a movable joint such as the one built on the splendid masonry dam for Pasadena, Calif., on the San Gabriel River.

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[48] *Transactions*, Technical Soc. of the Pacific Coast, Vol. II,

The other earthquake effect mentioned by Dean Morris is that of the earthquake waves that move outward from the rift and which are of sufficient energy to shake a large part of the earth surface with sensible effects. It is true that such waves might cause some settlement in a rock-fill dam. Two factors determine the magnitude of the effects. One is the intensity of the waves and the other, and more important, one is the nature of the foundation material upon which the dam rests. There has been no case that has come to the writer's knowledge where a rock-fill dam has been close to the origin of the waves.

In the California earthquake of 1906, the Sierra Nevada was shaken, but the waves were dampened by the granite core of the mountains. There were several rock-fill dams in the mountains subjected to the waves (Items 1, 2, 3, 10, and 11 of Table 1, among others) but no damage was reported on these dams. They were situated about 120 miles distant from the earthquake rift on the San Andreas fault. It is believed that the behavior of the earth dams located close to the fault line will convey some information of the behavior of a rock-fill dam. A description of the earthquake effect on several earth dams is given in the report to the Society by the Committee on the Effect of the Earthquake on Water-Works Structures.[49] With a few minor cracks, the dams were uninjured, and the same is true of the large concrete dam of the Crystal Springs Reservoir, situated within a few hundred feet of the fault line.

The foundation material is of the utmost importance in determining the movement of a dam in an earthquake. The amplitude of the earth movements may vary from less than 1 in. in solid rock to as much as 12 in. in loose alluvial soils saturated with water. As the rock-fill dam should only be built on solid rock, or material that approximates solid rock, the dam will not be subjected to much movement. In practically all instances of rock-fill dams that have come under the writer's observation in the Sierra Nevada, the foundation material is granite.

The settlements cited by Dean Morris, of fills on alluvial soils, mine dumps, and débris on steep mountain sides, do not represent the situation of a rock-fill dam located on bed-rock between two canyon walls. Mine dumps are usually found on side hills or in valleys with a cover of soil of some thickness. They are of small-sized rock, mixed with fine material. They would probably be shaken by an earthquake and some settlement would likely occur. The same is true of the miscellaneous débris on a mountain side that might be disturbed in such cases. The small earth dam that failed at Santa Barbara was located on an alluvial soil and was undoubtedly badly shaken, as the earthquake rift passed close to the dam.

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[49] *Transactions*, Am. Soc. C. E., Vol. LIX (December, 1907), p. 245.

It is the opinion of the writer that properly built, rock-fill dams, situated in mountains where a solid rock foundation is available, will not be distorted to a dangerous extent. The loose rock, when in place, is difficult to displace, excepting possibly on the down-stream face. This opinion is based upon the nature of the structure, the behavior of earth dams in an earthquake, and the fact that such dams as have been subjected to earthquakes have not been injured. However, the subject is an important one and the questions raised by Dean Morris are very pertinent. Until more information is obtained, the subject must remain an open one.

Mr. Peterson's statement regarding the rock used at Salt Springs Dam agrees with the idea that the writer desired to convey when the statement was made that individual rocks of fairly uniform size are desirable. More than one-half the rock was somewhat over 10 tons in weight; the excess fines were wasted, and as most of the fines that reached the dam were sluiced into the interstices, the rock that forms the main part of the dam was of fairly uniform size. It is a matter of description. The writer must differ from Mr. Peterson in the use of water in sluicing. When rock from the quarry is dumped into the dam, the small rock and fines will accumulate at the top. Merely to allow water to run into the mass will not carry the fines into the dam as well as to have a stream of water from a nozzle under pressure directed against the mass. The water will then break up the fines and force them into the rock. If this is not done, the fines will accumulate as a layer and will not be properly sluiced into the dam. The writer agrees that the volume of water is very important. In addition to moving the fines to position, the water acts as a lubricant, assisting in the necessary settlement.

The data, given by Mr. Howson, of three important dams with which he was connected, are of value as indicating methods of construction and the resulting settlement. It should be noted that the vertical settlement of Dix Dam, after eleven years, is slightly less than 1% of the height, which represents an excellent fill.

Mr. Harza raises an important point regarding the adaptability of a rock-fill dam to be used to create head for a power plant. On account of the nature of the rock-fill dam, repairs to the facing are generally necessary, whether the facing is of wood, concrete, or steel. This makes it necessary to draw down the reservoir. The writer is in complete agreement with Mr. Harza that the rock-fill dam is not a satisfactory type to be used for the purpose of creating head for a power plant.

In practically all dams of the rock-fill type, the reservoirs formed by the dam are used for storage only, the water being drawn from the reservoir to augment the low-water flow of the stream. At Salt Springs there is a relatively small hydro-electric power plant that operates by the flow of the stream when the reservoir is full or when water is being drawn from storage. However, the real function of the reservoir, at an elevation close to 4 000 ft above sea level, is to provide water for a chain of power plants on the river below the reservoir.

Below the plants, the water is used for irrigation. Sabrina and South Lake Reservoirs (Items 13 and 14, Table 1) also provide water for a chain of power plants, after which it is used for irrigation. The reservoirs are at elevations close to 9 000 ft. Bucks Reservoir, at Elevation 5 160 above sea level, performs the same function for power and irrigation. Some of the reservoirs designated in Table 1 are used for irrigation. It is believed that rock-fill dams, in general, should be used to form storage reservoirs only.

Mr. Huber has described in detail the timber facing of the Sabrina and Hillside or South Lake Dams on Bishop Creek. These dams emphasize the merits of timber facing. On the dams as originally built, there was only a thin layer of dry rubble. The timber facing conformed to the settlement of the loose rock-fill without damage and when the facing was replaced, after twenty years of service, the new timber facing was placed against a fill in which the major settlement had taken place. The new facing may be expected to serve for many years.

Mr. Fowler has added some interesting data on rock-fill dams in regions outside the United States and has given some comment upon the evolution of the type in California. Mention is made of the Relief Dam in this connection. Relief Dam can hardly be cited as illustrating one step in the evolution of dams in California. It was designed by the Consulting Board with a section different from the one built. There was no need for the large section of dry rubble, and as the work was done after the Board was no longer employed, it represented only the ideas of men unfamiliar with California practice. The evolution of the type used in California begins, as stated in the paper, with the dams on the Yuba River, notably, the Bowman Dam, which was built over thirty years before Relief Dam. In the evolution of the type, the principal change has been to abandon the rubble facing of the down-stream slope and to increase the thickness of the rubble on the up-stream slope. The face slopes of recent dams have also been made with flatter slopes. At Relief, the major advance was the introduction of a concrete face.

The Cogoti Dam, in Chile, was mentioned as the one where the facing was laid directly on the loose rock-fill without an intermediate cushion of dry rubble. The Sabrina Dam, built over twenty years previously, was substantially of this type, the facing being of timber resting on a thin layer of rubble that was little more than an arrangement of stones to a uniform surface. It is illustrated in the paper, and additional information may be found in Mr. Huber's discussion.

The behavior of the laminated facings of the Algerian dams throughout the years will be of considerable interest. Contrasted with the California dams, the design of which is based upon the experience of over sixty years, the Algerian dams represent a radical departure. The writer may be allowed to express doubt regarding the permanence of laminated facings built of asphalt concrete, especially where it is necessary to spray the face with water during hot weather. It might happen that on some warm day the water spray will be omitted.

It may be questioned whether a citation of the design of dams built in different parts of the world should be given comparable weight with those dams which have had a considerable life history and in which time has been given to develop such defects as may occur during service. The dams cited by Mr. Fowler, in Algeria, Chile, and in China, are of recent construction. Information regarding their behavior during a few years of service will be of interest and may form the basis for conclusions differing from the ideas held when the dams were designed.

Messrs. Steele and Dreyer have contributed, in their excellent joint discussion, the most important data on the settlement of a large rock-fill dam that has ever appeared; that at Salt Springs Dam. The measurements extend over the five years of the life of the dam and (what is of great value) the measurements of settlement apply to areas of the dam face below the crest. It should be noted that the settlement under the combined rock weight and water load is not a maximum at the crest but is found on the slope of the dam. Settlement due to the weight of the rock occurs progressively during construction and will continue for years after the dam is placed in service. It is difficult to obtain the cumulative effect by direct measurement. The settlement under water load can be determined as stated in the discussion. Above the level of the main quarry (Elevation 3 805), there was a fairly close agreement between the calculated settlement when the design was made and the measured settlement. Below that elevation the settlement was considerably greater than that assumed. It may be doubted whether any exact calculations can be made, as so much depends upon the nature of the rock and the methods used in the construction. However, the theoretical treatment is of value as indicating what may be expected at any time when the crest settlement is known or assumed. In time the crest settlement will increase and the face of the dam will tend to straighten.

The necessity for additional thickness of rubble at the bottom and on the sides of the canyon walls is well stated, as is also the connection of the concrete face to the cut-off wall. The statements represent the experience of the engineers who have been charged with the care of the dam. The writer is in agreement with the conclusions expressed in the discussion.

Mr. Sanger has presented some data on the failure of the Eildon Dam, in Australia, and Mr. East has given a more extended description of the regrettable failure. The writer had occasion to discuss the paper by Mr. Knight,[50] and certain statements were made in that connection.

The presence of clay in the bed of the stream indicated that the material should have been removed. It may have been an important factor in the horizontal movement of the up-stream section of the dam. The concrete core-wall acted as others have done. Under the water load it deflected down stream over considerable lengths, the maximum movement at the top, to 1933, being over 7 ft. The thickness of the concrete core-wall was sufficient to have held the water, but there was added a vertical wall of clay, averaging 30 ft thick, an unnecessary feature. The rupture and movement of the up-stream rock-fill were undoubtedly due to the distortion of this clay, aggravated by the rock being squeezed into the clay under its own weight, and to the lubrication of the clay when submerged.

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[50] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by R. G. Knight, *Journal*, Inst. C. E., March, 1938.

The writer is in agreement with the conclusions expressed by Mr. East except that the fracture of the core-wall could not be attributed to the pressure of the clay wall in front of it. Even if the clay wall had been absent, the movement of the rock mass under water pressure would probably have produced the same result. It is one more example of a failure showing that a core-wall in the center of a dam is a feature to be avoided.

Messrs. Gomez-Perez and Jinich have described in detail the building of five important rock-fill dams in Mexico, the design of which follows, in general, that of California dams. Some comment may be offered on certain features of design.

There seems to be a tendency in the later dams to use a flatter slope on the up-stream side, the reason being that, on the whole, this form is cheaper. The dams were built in layers of relatively small height. There seem to be good reasons why thicker layers would be better, as referred to later. Opinions will differ on this idea, as there are arguments to be presented on both sides. The writer is impressed with the small dimensions of the facing slabs, as experience elsewhere indicates that larger slabs can be used. In general, it would seem that the Mexican engineers have built dams that are adapted to the location, where there is cheap labor and where the importation of expensive machinery was not possible.

Mr. Reed has presented interesting data regarding San Gabriel Dam No. 2—the causes and results of the partial failure and the destruction and removal of the laminated concrete facing.

Other discussers have contributed important data and information of a general nature that does not call for detailed comment. There are certain other elements referred to in the several discussions, upon which the following observations are made.

A number of those who discussed the paper have disagreed with the writer regarding his statement that no gates of any kind should be placed in the spillway. The conclusion, as expressed, was reached after a study of dams that have been destroyed, either earth or rock-fill. The first principle that must be borne in mind is that safety of the structure must, at all times, be assured as far as it is humanly possible to do so. The danger to life and property that exists on every stream below the dam is the major consideration that should guide an engineer in making the design. It is true, as Mr. Fox states, that rock-fill dams are very stable and will resist some water over the crest without failure. Bear River Dam (Item 10, Table 1) was overtopped in one flood without failure. However, three fairly large dams were destroyed by being overtopped, due to inadequate spillways: Walnut Grove, Lower Otay, and Castlewood Dams (Items 4, 5, and 8, Table 1). It is obvious that a sufficient quantity of water over a rock-fill dam will wash it out, and that in three cases at least, this destruction has occurred.

It will be argued that the gates will be opened in time of flood and thus preserve the dam. There are reasons why the gates may not function at the proper time. The operating force will grow negligent and will fail to draw down the reservoir at the proper time, if at all. Storms and resulting floods are sudden, especially in mountain areas where a warm rain falling on snow will bring down a maximum flood in a few hours. Again, rock-fill dams, as a rule, are built in remote locations and usually only one or two men are in attendance at the dam. The failure of a power line, or the failure or inability of the attendants to operate the assumed gates, may cause the overtopping of the dam.

A further argument is found in the fact that the exceptional flood will bring into the reservoir trees and brush that have accumulated over the years, many of large size. This is a common result of the exceptional flood, noted in many instances. The débris will accumulate against the gate structures, in spite of fender logs. The spillway thus clogged will fail to function and the water will go over the dam. Many years ago the writer was connected with the rebuilding of an earth dam that had been washed out due to clogging of the spillway by débris brought into the reservoir in the flood caused by an exceptional storm. The spillway was formed by openings provided with gates placed between masonry piers. It had functioned for over fifty years without damage but failed in the exceptional storm and flood and the dam was washed out.

In a storage reservoir created by a rock-fill dam there is no need for gates. The dam and the reservoir capacity can be ascertained from a study of stream flow and the requirements of storage. The spillway can be located at the proper elevation and gates to regulate the elevation of water are unnecessary. However, the major factor in reaching a decision is found in the fact that rock-fill dams have been, and can be, destroyed by being overtopped by water. Gates in the spillway have failed to function at times, and since no effort should be spared to make the structure safe in view of the possible loss to life and property, an open spillway of large size, capable of passing all floods with attendant débris, should be provided at all dams. The writer emphatically repeats the statement of the paper that no rock-fill dams should have a spillway dependent upon gates of any kind.

The subject of settlement requires further explanation. The statement of the writer that settlement might be 5% of the vertical height referred to settlement of the loose rock-fill during construction and before the building of the rubble cushion and the impervious face. This was made to emphasize the necessity for deferring the construction of the other two elements of the dam for as long a period as possible after placing the loose rock-fill. If the dam has been properly constructed, the final settlement may be less than 1% of the height, although it will continue to settle for a long period of years. Much depends upon the manner in which the fines have been sluiced into the dam. An extreme case is found in San Gabriel Dam No. 2, where no sluicing was done during construction. During the first heavy storm the dam settled about 12 ft, wrecking the facing. The data of settlement at San Gabriel Dam No. 2, given by Mr. Baumann, seem to refer more to the relation between the quarried rock and the volume within the dam. In general, the settlement discussed herein refers to the settlement during construction and especially after the completion of the dam.

The problem of settlement is one of the most difficult to solve in the construction of rock-fill dams. The higher the dam, the more important becomes the problem. Settlement is intimately connected with the methods used in construction. In some dams, as in Algeria, the engineers have gone to the extreme of making the entire dam of rubble. In others, the loose rock-fill has been deposited in relatively thin layers. There are reasons for so doing, but there seem to be better reasons for the construction of the dam in deep layers. The loose rock, dumped at the top of a fill (say, 100 ft in height), rolls down the slope and by its impact, beds itself among the other rock. It does not merely rest upon a point, because the rock is crushed to a bearing. The fill in such a case is not merely a mass of rock loosely dumped together. The rocks are wedged among each other and form a compact mass, differing greatly from a loose mass. It is quite true that the smaller rock accumulates at, or near, the top of the slope, but the general effect is a compacted mass.

On the other hand, the definition of rubble contains a wide variety of ideas, which are difficult to describe. It is quite possible to build such a wall that will have but little settlement and also to build one that will act in the reverse manner. Merely to place the rock by hand or derrick will not necessarily produce a compact mass. The final results may furnish a means of comparing the loose rock-fill with one of rubble. In Algeria the Ghrib Dam was built entirely of a rubble fill, and a settlement of 0.8% of the height took place, presumably in the first year, while on the contrary, the loose rock-fill at Salt Springs had settled at the crest, only 0.6% of the height in five years. In eleven years the Dix Dam has settled less than 1.0% of its height. The sluicing of the fines into the mass of the dam is an important factor in reducing the total settlement.

The position of the impervious element seems to be agreed on, but some dams have recently been built with the element placed as a vertical wall in the center of the dam. At the Eildon Dam, the top of the vertical core-wall, about 140 ft high, was moved down stream, a distance of about 7 ft as a maximum when the reservoir was filled. At one of the dams in Tunis, with a hollow concrete central wall, about 115 ft high, the crest moved down stream about 2 ft 8 in. on the first filling of the reservoir. The wall was ruptured. The plan of using a central core-wall seems to have arisen from the idea that the weight of the rock-fill is vertical and ignores the effect of water pressure. The actual forces are made up of the rock weighing about 100 lb per cu ft, which acts vertically, and the weight of water at 62.5 lb per cu ft, which acts horizontally. The two forces are of the same order of magnitude and the resultant causes a compression of the material of the dam in a direction that is represented by the resultant of the vertical and horizontal forces. The movement off the core-wall down stream and its possible rupture may be foreseen.

The material of the impervious face was described. In the case of a concrete facing it is believed that a laminated face is unnecessary and will tend to be ruptured. At San Gabriel Dam No. 2 the laminated face was badly ruptured, and it was necessary to install a wooden facing to take its place. It should be recognized that the concrete facing is a very thin element compared to the dimensions of most dams. It is subjected to many forces during settlement which requires that it be of sufficient thickness of integral concrete to withstand distortion. A laminated facing will not perform this function to any great extent. Although much of the discussion has centered around concrete as a material for facing, the writer is of the opinion that timber is an excellent material for the purpose. It will follow the distortion due to unequal settlement and may be renewed, as mentioned by Mr. Huber in his description of the renewal of the facing of Sabrina and Hillside Dams. In this remote location, a timber face is cheaper and is all that is required. The writer also believes that a steel facing has similar good qualities to recommend it. It is permanent and the cost will be a determining factor.

The discussion has centered around some of the higher dams, of which Salt Springs, with a height of 328 ft at the center, is the highest yet built. The higher the dam, the greater the total settlement. The rigid facing, resting as it does on the rock-fill, must move during the settlement. At the bottom and sides of the canyon it must be attached to the rigid cut-off wall. The result is that expansion joints must be installed. Their design is difficult. The comment of Messrs. Steele and Dreyer on the experience at Salt Springs illustrates this point. Although individual situations will always determine the height and selection of the type of dam, it is the belief of the writer that rock-fill dams of a height approximating 200 ft represent about the advisable maximum.

The writer desires to express to those who have discussed the paper his appreciation of their kindness in so doing and for the data given. It is believed that the entire discussion will furnish information of value to those who may have before them the problem of designing dams.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

Discussion

BY A. M. RAWN, M. AM. SOC. C. E., A. PERRY BANTA, ASSOC. M. AM. SOC. C. E., AND RICHARD POMEROY, ESQ.

A. M. RAWN,[23] M. AM. SOC. C. E., A. PERRY BANTA,[24] ASSOC. M. AM. SOC. C. E., AND RICHARD POMEROY,[25] ESQ. (by letter).[25a]—Replying to Mr. Moore's question about water temperature in the heating coils: When the rate of heat conductance was 13 Btu per hr, per sq ft of pipe surface per degree Fahrenheit of temperature differential, the test records show that the temperature of heating water into and out of the heating coils was 180° and 143° F, respectively. After the coils were cleaned and the conductance rate was 54 Btu, the temperatures were 151° and 113° F, respectively. The writers' observations confirm Mr. Moore's comment that higher temperatures of the circulating water accelerate lime deposit on the coils. Design data for the District tanks contemplated a maximum temperature in the coils of 135° F, but the tanks are handling more than twice the sludge originally contemplated, and it is necessary to raise the circulating water temperature correspondingly.

The writers deemed it impractical to attempt to derive coefficients of heat loss through the outside tank walls, as suggested by Mr. Moore, because the gross heat loss of 1 200 000 Btu per day includes, inseparably, the radiation losses and the endothermic (or exothermic) requirements of the bacterial process. The mild climate of California further complicates the problem.

Mr. Keefer questions the temperature measurements. The quantity of heat added to the sludge by the heating water was determined by metering the

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NOTE.—The paper by A. M. Rawn, M. Am. Soc. C. E., A. Perry Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, Esq., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1938, by Edward W. Moore, Esq.; March, 1938, by Messrs. Willem Rudolfs and H. Heukelekian, and Hermann Bach; and September, 1938, by C. E. Keefer, M. Am. Soc. C. E.

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[25a] Received by the Secretary October 10, 1938.

flow, and by measuring the temperature of the water entering and leaving the tanks, with a mercury thermometer that could be inserted into the stream at several locations. Temperatures were read to 0.5° F and averages of sixteen days were taken. It is believed that the error in the value for heat input is probably less than 3 per cent. Temperature of the raw sludge was assumed to be the same as sewage temperature, the latter being read three times daily.

Temperatures in the tanks are read daily to the nearest degree by means of thermo-couples placed near the middle of one of the side bays of each tank. In the first tank there is some uncertainty as to the values due to local temperature differences, as presented by Mr. Keefer; consequently, these values are not relied upon in attempting to ascertain the heat effect of the process. However, the differences are not great, as Table 7 shows. When the measurements in Table 7 were made, readings were also taken at three depths in the west bay of Tank No. 1, Battery A, at a distance of 1 ft from the heating coils, giving values of 85.2°, 85.2°, and 86.8° F. The average is only 2.1° higher than at the middle of the bay, accounted for by agitation due to vigorous digestion, and to the absence of extensive stratification of solids at that time (see Fig. 4(*b*)).

Concerning the second, third, and fourth stages, the rapid flow of the sludges, the vigorous digestion, and the fact that no heat was added and little was lost, are reasons for believing that the temperature must have been quite uniform throughout those tanks. Hence, the routine thermo-couple readings are probably representative of the average temperatures. Sixty-six readings in the third and fourth stages, taken during the period of the heat-effect study, all showed 85° F. In the second stage there were twenty-seven readings of 85° F, four of 86° F, and one of 87° F. Thus, the indicated loss from the second to the fourth stage was 0.19° F; but since a consistent effect as great as 0.5° might escape notice, it was reported that the fall in temperature "was less than 0.5° F."

The estimated difference in free heat between the raw sludge and the digested sludge depends upon the raw sewage temperatures, the temperatures in the fourth tanks, and the measurements of the quantity of sludge pumped. The latter quantity was obtained from pressure readings, and thus represents the actual weight of sludge pumped, free from any error due to entrained gas. Seeding had no effect upon these calculations, since seed was taken from the fourth tank. It is believed that the error in the free-heat difference is probably less than 10 per cent. On the basis of experimental error, it would be very difficult to account for the difference between the possible endothermic effect in this digestion system and the results of Messrs. Keefer and Kratz.

It was not intended that any question should be cast upon the validity of the results reported by Messrs. Keefer and Kratz. As has been noted in the discussions by Mr. Moore and by Mr. Keefer, different behavior in respect to the heat balance may be expected from different sludges. The data were presented to contribute some further information obtained under different conditions. Unfortunately, the data do not allow an exact evaluation of the heat effect, since the heat lost to the surroundings cannot be determined.

The results do show that the process was not accompanied by an endothermic effect as large as that observed by Messrs. Keefer and Kratz.

The writers wish to minimize the significance of the greater weight (7%) of gas produced when compared with the weight of organic matter destroyed. The observational basis of the indicated difference is weak and, therefore, the experimental error may be relatively large. Furthermore, this value (7%) takes no account of the carbon dioxide, ammonia, and volatile acids lost from samples of new or digested sludge upon drying during analysis. The theoretical basis of such a difference is well established but the derived value (7%) should not be considered a precise empirical value.

Regarding scum accumulation, Messrs. Rudolfs and Heukelekian state: "If the digestion time is insufficient the mass of scum eventually becomes so large that the capacity of the tanks is reduced materially and the scum must be taken care of by other means." That this situation does not apply to the installation under discussion is witnessed by the digestion experiments conducted on samples of the scum. It was found that, for the first four days after removal from the tank, scum from the first stage produced gas at a rate of 5.0 liters of gas per liter of scum, whereas the highest measured rate of gas production from the first stage taken as a whole was 3.11 liters of gas per liter of sludge (see Table 5). It seems quite likely that after twenty-five months of operation, conditions regarding scum accumulation may be considered static (see Fig. 4). Tanks were emptied because of the condition of the heating coils, not because of scum accumulation.

No pronounced difference has been noted in the drainability of sludge from the tanks and from the pit. Detail results are at hand relating to fourteen beds of sludge drawn during the period of these tests. Five beds from the pit, averaging 4.49% solids, lost from 16% to 37% of their water in 16 hr, the average loss being 28 per cent. Nine beds from the tanks, averaging 4.77% solids, lost an average of 26% of their water in the same length of time—the slight difference being in the direction that would be expected from the variation in solids content. No explanation is apparent for the erratic variation of individual beds.

Exactly at what point a sludge is "well-digested" is a moot question. The writers believe that a well-digested sludge is one that drains and dries without attendant nuisance, in a reasonable length of time. Freedom from nuisance in drying seems to be closely connected with the content of volatile organic acids. Experience at the plant under discussion indicates that sludge that has less than 200 ppm of volatile acids can be placed on the drying beds with positive assurance that it will not breed house flies and that there will be no objectionable odors. As for drainability, it is doubtless true that, in general, prolonged digestion will produce a sludge which drains rapidly. However, in a separate sludge digestion system it is not economical to aim for the ultimate in drainability. Sludge beds are usually cheaper than digestion tanks.

In Mr. Bach's discussion of the optimum relative size of stages in a digestion system, the writers do not concur in the desirability of proportioning the stages so that an equal volume of gas is produced from each stage. Admittedly, there is little technical information on which to base the choice and, in the installation under discussion, judgment and operating and structural considerations have dictated.

Mr. Bach has misinterpreted Column (7) of Table 6. This column was meant to indicate that, of all the gas produced in the digestion tanks, plus the gas produced with 21 days additional detention time, about 83% was produced in the tanks and 17% was produced in the 21 days of additional detention time.

Scum at this plant contains entrained sludge particles, seeds, matches, hair, bits of rubber, and other light débris. The fact that the moisture content is only 85% and that the surface is in contact with sludge gas does not seem to inhibit active digestion as is suggested by Mr. Bach. That it is the seat of active digestion has been demonstrated previously. The sludge gas is saturated with moisture and has no drying effect.

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DISCUSSIONS

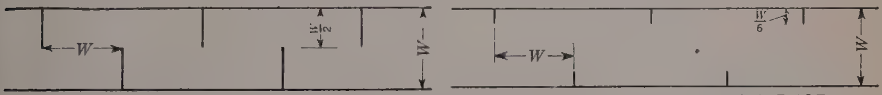
OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

Discussion

BY E. H. TAYLOR, JUN. AM. SOC. C. E.

E. H. TAYLOR,[6] JUN. AM. SOC. C. E. (by letter).[6a]—The fact that geometric distortion of river models is theoretically invalid has long been recognized. Solution of the model equations for a number of effects—for example, backwater, tractive force, or transverse slope at a bend—leads to the conclusion that $L_r = d_r$, or that the distortion ratio should be unity. Tests conducted at the Hydraulic Laboratory of the University of California at Berkeley, Calif., during 1935 and 1936 support Recommendation (1) that "geometrically distorted river models should be avoided whenever suitable undistorted models can be constructed at reasonable cost."

In an attempt to determine the effect of geometric distortion upon the bed configuration of a movable bed model, an arbitrary prototype was constructed in a channel 1 ft wide, and its model was built in another channel to a horizontal reduction scale of two. The model was constructed so that it could be operated as either a distorted or as an undistorted model. Arbitrary rates of flow were set up in the prototype so as to produce definite bed configuration about certain simple obstructions which could be easily modeled in the smaller channel. The obstructions, or baffles, about which the flow was made to pass, were built to cause either a high degree of turbulence, or to allow the flow to proceed smoothly. Shown in Fig. 6 are plan views of two of the arrangements used, in which w = channel width (either 12 in., or 6 in.).



(a) HIGH DEGREE OF TURBULENCE (b) LOW DEGREE OF TURBULENCE

FIG. 6.—ARRANGEMENT DESIGNED TO PRODUCE TURBULENCE

NOTE.—The paper by Kenneth D. Nichols, Jun. Am. Soc. C. E., was published in June, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Herbert W. Ehrgott, Esq.

[6] Instructor, Mech. Eng., Univ. of California, Berkeley, Calif.

[6a] Received by the Secretary August 22, 1938.

No attempt at "verification" of the prototype bed configuration was made, but discharges and lengths of time for the model were calculated on the basis of the Froudian law of similarity. At the conclusion of each test, the elevations of the prototype and model beds were compared at a large number of corresponding points and an error curve was prepared. Fig. 7(*a*) shows the relative correspondence of the distorted and undistorted model to their prototype obtained from a typical set of runs. The agreement between models and prototypes was found to be superior for the condition represented by Fig. 6(*a*).

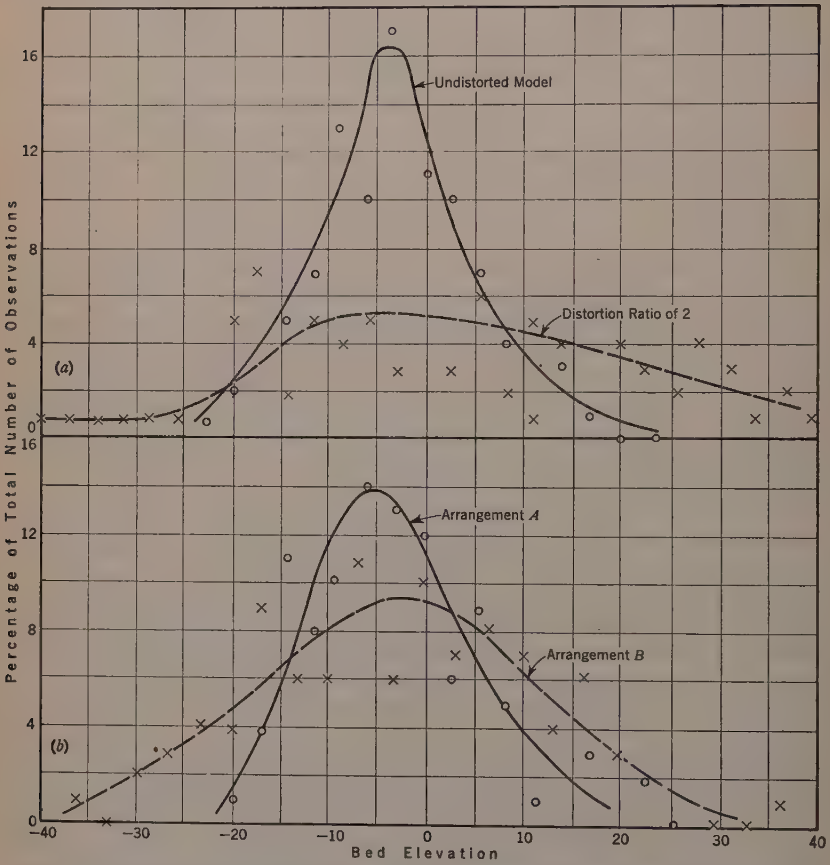


FIG. 7.—VARIATION OF BED ELEVATION AT CORRESPONDING POINTS OF MODEL AND PROTOTYPE IN PERCENTAGE OF MAXIMUM DIFFERENCE IN BED ELEVATION

Fig. 7(*b*) shows the relative correspondence of undistorted models of Arrangements *A* and *B* to their respective prototypes. It was concluded that:

(1) Undistorted models more truly represent their prototypes than distorted models; and,

(2) If Froude's law is used as the model criterion, models of highly turbulent systems are more faithful than models of relatively smooth flows.

The writer's understanding of "model verification" of movable bed models is as follows: A known hydrograph covering a certain period of time in Nature is simulated in the model. Several trials may have to be made, adjusting the model discharge, slope, or time, until a certain combination of hydraulic conditions is obtained whereby the model reproduces the natural bed condition. The model is then said to be adjusted, and the assumption is made that these experimentally determined hydraulic conditions fix the proper ratios for discharge, time, etc., for the particular model. The model is then assumed to be capable of indicating future changes in bed conditions which would be due, perhaps, to proposed training works.

It may be reasoned, however, in view of the foregoing Conclusion (2) that the degree of turbulence, or nature of the flow system, is a factor that influences the discharge or time scale necessary to produce similarity of bed configurations between a model and its prototype. Thus, the introduction or removal of groynes and dikes in the model stream would effectively alter the turbulence which would seem to invalidate the experimentally determined model ratios. The matter of movable bed model "verification" impresses the writer as being somewhat uncertain.

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DISCUSSIONS

THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD

Discussion

By Messrs. Oscar S. Adams, and George D. Whitmore

Oscar S. Adams,[2] Esq. (by letter).[2a]—Many papers dealing with the three-point problem can be found in periodicals devoted to engineering and surveying. Any treatment of the subject by means of co-ordinates must necessarily depend upon the same principles. The paper by Mr. Rowe is developed in a very complete manner and it should be useful to any surveyor who may wish to take advantage of this method of connection to control.

The U. S. Coast and Geodetic Survey is following the practice of computing the plane co-ordinates of all the triangulation stations on the State plane-co-ordinate systems. In the end every station will have its plane co-ordinates given as well as its geodetic position. For some States this work is already practically completed and, in course of time, this will be true of all the States. In any region that lies near the various arcs of triangulation, tanks, steeples, and other such objects, are located by intersection observations. This work should form a good basis for the application of the three-point fix.

Besides the work of the Coast and Geodetic Survey, the U. S. Geological Survey, the Corps of Engineers of the U. S. Army, and other surveying and engineering organizations have located many prominent objects in the vicinity of their operations. All these stations can be used indiscriminately if they are all computed on the same basis and are mutually dependent for their datum. Of course if they are not so co-ordinated, each would have to be used independently of the others. The Coast and Geodetic Survey developed the State plane-co-ordinate systems with the hope that all other organizations would make use of them in their work and that all work would be tied to the control net of the country. If this hope should be realized, all work of the various organizations would be on a common basis, and all results would have a permanent value for any succeeding work. If this could be accomplished

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Note.—The paper by R. Robinson Rowe, M. Am. Soc. C. E., was published in June, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[2] Senior Mathematician, U. S. Coast and Geodetic Survey, Washington, D. C.

[2a] Received by the Secretary July 28, 1938.

there would be a great mass of data available for use in many regions where now there is great lack of such control.

The three-point fix has a practical and valuable use in hydrographic surveying, in determining locations offshore, and in plane-table work in topographic mapping. A three-point location, however, even when dependent on accurately located stations, is not of sufficient accuracy to justify its use in the extension of control, except for surveys of the lower order. The use of intersection stations is not advocated as the basis of control for second-order surveys and little dependence can be placed, as far as control is concerned, on a three-point location which is based upon tanks, steeples, water towers, or intersection stations of any kind.

In army operations in time of war, much use is made of this method of determining positions. The Allied Armies in France had a method of computation for the three-point fix dependent upon the grid system in use in the World War Zone. This method, of course, depended upon plane co-ordinates.

The author is to be congratulated upon the excellence of his development of the computation formulas. The treatment is logical and the resulting formulas would be easy of computation with an ordinary calculating machine. The discussion of the accuracy of the results is very well done. As a final word, it is desired to repeat the warning against the use of this method where any accuracy of location is essential. As already stated, it has its use—and an important use—but as control for further work, it is not to be employed even under the most favorable circumstances.

Corrections for *Transactions:* In Equation (18) numerator should read "$(\delta'')\ ab$", not "$(\delta'')^{ab}$"; and in Fig. 3, the scale in right-hand corner should read "Scale in Hundredths of Feet."

George D. Whitmore,[3] M. Am. Soc. C. E. (by letter).[3a]—As shown by the author, three-point triangulation has a definite place in modern surveying and mapping practice. For its successful use, however, its peculiar characteristics and limitations must be recognized. For example, it may be said that, in general, a horizontal position established by three-point triangulation (resection) is never as strong as the position established from the same basic control stations by "cuts" (intersection). Furthermore, it is true that there are many instances in which the position from a three-point determination might be uncertain or even unsolvable, whereas the same position might be strong if the intersection method of "cuts" were used. In many cases, the strength or weakness of a three-point location is not immediately apparent to the field man, this being a rather serious disadvantage if the basic control stations are scarce and if advance planning of the field work is not practicable.

Three-point triangulation can often be used to advantage, however, under the following conditions, if too high an order of accuracy is not required:

(1) When it is not practicable to occupy the basic control stations (for example, if the basic control includes a large percentage of spires, cupolas,

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[3] Chf. of Surveys Section, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[3a] Received by the Secretary October 3, 1938.

stacks, towers, etc., impossible to occupy with an instrument for "cut-in" work without an expensive building for instrument support);

(2) When the basic control is already signaled (in this case, under the three-point method, no new construction would be required except for instrument supports at the three-point stations);

(3) When the ratio of new supplemental stations to existing basic stations is high (obviously, it would be cheaper to construct a few signals at the existing basic stations for three-point observations than to construct many signals at the new supplemental stations for "cut-in" observations); and,

(4) When flexibility in the observing schedule is required.

During the two years, 1937–1938, in connection with a topographic mapping project in the Tennessee Valley, more than 700 three-point stations have been established as detailed horizontal control. The terrain being mapped under this program includes several thousand square miles in the Appalachian Mountains of Eastern Tennessee and Western North Carolina, and is generally heavily wooded and quite inaccessible. There are many peaks with elevations that exceed 5 000 ft, the floors of the valleys averaging about 1 200 ft. The basic control already in existence consists of the first-order and second-order triangulation of the U. S. Coast and Geodetic Survey, with about one station to each 50 sq miles; and third-order triangulation of the U. S. Geological Survey, with an average of one station for each 12 sq miles. It is necessary to supplement this basic control with additional stations averaging about one for each 4 sq miles. Three-point triangulation is the principal method used to establish this supplemental control.

The mapping of this area is by the stereo-topographic process, using aerial photographs. Because of the characteristics of this method, the supplemental control stations must be established in certain restricted locations, namely, in the common overlap areas of several photographs, and close to some definite point which can be identified on all the overlapping photographs. Thus, there is very little latitude in selecting the locations of these supplemental control stations. Furthermore, the observing schedule is such that it is not practical to "cut in" all supplemental stations with one visit to each basic control station. If the "cut in" method were used, many return visits to these basic control points would be required. Under the three-point method, only one trip is made to each basic control station, at which time specially designed pyramid signals are erected, and precise vertical angles measured for the determination of elevations on these basic stations.

In order to meet the accuracy required by the maps, the horizontal position of each supplemental control station must be correct to within 10 ft in relation to the basic control, the horizontal plotting scale of the map being either 1 : 10 000 or 1 : 15 840. To obtain this fourth-order accuracy in the three-point locations, repeating theodolites are used, equipped with verniers reading to 0° 0′ 20″. Six repetitions direct and reverse are taken on each angle, including the angle closing the horizon. The lengths of the sights occasionally run as high as 10 miles, and average 4 to 5 miles. Usually four or more basic control stations are visible from each three-point station, all near-by visible basic stations being observed, of course.

Early in this program the advantages of making all computations and adjustments of three-point stations in a co-ordinated field were recognized. The State-wide plane co-ordinate systems originated by the Coast and Geodetic Survey were adopted as the computing medium. It is interesting to note that the formulas developed by the computing staff for the solution of the three-point locations are substantially the same as those developed by the author. A simple and "fool-proof" computation form, based on Equation (10), has been in use for some time. The angular value of Z is obtained by an inverse solution as a part of the routine check on the work. The research in developing these computing and adjusting methods was done principally by W. O. Byrd, Assistant Mathematician, working under the close direction of C. C. Miner, Assoc. M. Am. Soc. C. E., head of the Surveys Computing Staff of the Tennessee Valley Authority.

The formulas developed by the author to determine the "relative weight and figure strengths" are very useful and have an important application. For the successful utilization of three-point triangulation, it is absolutely necessary to avoid weak figures. The Surveys Computing Staff has made extensive use of this principle, not so much in the refined adjustment of the three-point figures, as in selecting the strongest figure from which to establish each three-point position.

The experiences of this same organization indicate that a refined or delicate adjustment of a three-point position is not justified unless the basic control is of a high order of accuracy, and located so as to provide several reasonably strong figures for each three-point location. When these conditions do not exist, the method used has been to select the strongest figure and base the determinations entirely upon that figure. The remaining pointings are then used as checks on the work. These checks are made rapidly and consist of comparing the observed directions with the computed directions.

Occasionally, it has been found desirable to determine the most probable value of a three-point position by taking advantage of all observations to all basic stations. The least squares method of adjustment,[4] is very satisfactory and can possibly be made in less time than the "cut-and-try" method advanced by the author. An experienced computer can complete such an adjustment with pointings on five stations in less than 2 hr.

The method of establishing elevations at three-point stations in connection with the topographic mapping project may also be of interest. Elevations are required to be correct to within 4 ft, with respect to the nearest basic vertical control, the contour interval being 40 ft, with half-interval, 20-ft contours being shown in all flat and critical areas. Elevations are first established on the basic control stations, either by spirit leveling, or by vertical angles of a high degree of precision. To determine the elevation of a three-point station, vertical angles to each of the visible basic control stations are read, the measurement consisting of two readings with telescope direct and two readings with telescope reversed, using a vertical circle reading to 0° 0′ 20″. Then, utilizing the horizontal lengths computed from the three-point problem, the observed vertical angles, and a field-determined coefficient of refraction, the difference of elevation between the three-point station and each basic control station observed upon is computed. Each basic control station observed thus furnishes an elevation for the three-point station, the final elevation of the new station usually being the mean of the elevations from the various basic stations.

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[4] *Special Bulletin No. 138*, U. S. Coast and Geodetic Survey, p. 114.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WIND FORCES ON A TALL BUILDING

Discussion

BY MESSRS. DAVID CUSHMAN COYLE, AND CLYDE T. MORRIS

DAVID CUSHMAN COYLE,[9] M. AM. SOC. C. E. (by letter).[9a]—In the design of slender towers to be occupied by paying tenants, the proper criterion is not so much strength against being blown down as a feeling of solidity noted by the tenants during high winds. Few cases will be found in which the requirements of sensation do not give a tower of a highly conservative design as to strength.

The theory advanced by H. V. Spurr,[10] M. Am. Soc. C. E., is adaptable to a sensation criterion, since it involves forcing the floors to remain plane, thus facilitating the mathematics of cantilever vibration analysis, and since it also involves as an arbitrary constant the allowable static deflection.

For the purpose of analysis for sensation, the writer would modify the (wholly nominal) assumed wind load used by Mr. Spurr. Instead of 0 at the street line and 30 lb at the top, the writer would use $0.03\ h$ lb at the top, which is 30 lb at Elevation 1 000.

There are two assumed constants to be used in the proposed modification of the Spurr theory, both of which may be refined by research. One is the absolute allowable wind loading, a value to be used in checking the design to guard against overturning. Professor Rathbun points the way to a method of evaluating this constant. Obviously, it is not a real pressure per square foot, evenly distributed over the building; nor does it have a relation to anemometer readings that can be discovered by logical analysis. However, if a series of readings of static deflection can be obtained and correlated with anemometer readings, and if the stiffness of the towers thus treated is known, it is obviously possible to obtain a nominal wind load varying from 0 at the bottom to $f(a)\ h$ at the top, that would produce, theoretically, the observed deflection ($f(a)$ is a function of the anemometer reading).

The stiffness of existing towers must be measured in order to analyze the foregoing factors. Generally, the designer must deal with irregular spacings of

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NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in September, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order to bring the views expressed before all members for further discussion of the paper.

[9] Cons. Engr., New York, N. Y.

[9a] Received by the Secretary October 10, 1938.

[10] "Wind Bracing," by H. V. Spurr, McGraw-Hill, 1930.

columns, offsets, and other anomalies. The mathematical treatment of this problem was published by the writer in 1932.[11]

Using the observed period of vibration, and the estimated actual dead and live loads, the actual stiffness can be determined and expressed in assumed nominal wind load per inch of static deflection. This is a different kind of quantity from the wind load previously mentioned. That is, a maximum wind load of some kind will be imposed by Nature, and the only question in this regard is to find a nominal load which, when applied to the analytical formula, will result in a tower that will stand up. On the other hand, the stiffness is an intrinsic quality of the building definable as a ratio between load and deflection, and has no concern with whether a load of that form or size could ever exist in Nature.

Thus, by knowing the period of vibration—easily observed with a seismograph—and the weights, the true over-all stiffness of towers (including effects of steel and masonry) is easily found. Using this known stiffness, it will be possible, at considerable expense for research, to find, roughly, the proper absolute nominal wind load for design of stability.

Thereafter, a maximum hurricane of any specified velocity could be assumed as measured by the anemometer, and the wind diagram to be used in checking for over-turning could be derived therefrom.

There remains the problem of securing the proper stiffness for comfort. To obtain an analytical description of the criterion for comfort it is necessary to know, first, the form of the function of period and amplitude that corresponds to sensation. This function has been commonly assumed to be acceleration, which, since amplitude is proportional to static deflection, is described by the ratio, $\frac{y}{T^2}$, y being the deflection and T, the vibration period. The conclusion reached by Mallock that there has been some experimental work indicating vibration reaches the threshold of sensation at an acceleration of 1% of g. The vibrations involved, however, were tremors of the order of 5 to 25 per sec; the results, therefore, do not apply to wind sway. The writer has felt vibrations of an amplitude of about 0.5 in. at 13 per min, which gave an acceleration of only about 0.25% of g. It seems that the feeling of slow sway is different in kind from the feeling of tremor. It is possible that tremor may be a muscle or skin sensation, whereas sway may be a sensation of the semi-circular canals.

Another seeming fact throws light on this question. If a tower of given size and weight were to be made twice as stiff by the addition of extra bracing, the acceleration would not be changed because the amplitude would be cut in two and the period would be divided by $\sqrt{2}$, leaving $\frac{y}{T^2}$ unchanged. Adding stiffness to a tower without changing the weight, therefore, does not improve its acceleration; yet it seems fairly clear from observation that the stiffer tower is the more comfortable.

If the function is not $\frac{y}{T^2}$, it seems necessary to make tests on a shaking-table in a psychological laboratory to find out what the function is. The next step would be to measure a large number of towers for T and y, by means of the seismograph for T and analysis using the weights so as to find y. Then, using the necessary function of T and y, these towers may be arranged in statistical order, from the worst to the best. By observing this array one can then pick the desired flexibility or stiffness to correspond with the quality of building desired. Thus, a tower can be planned so that its amplitude and period are in a class with the Woolworth Building or the Metropolitan Life Building (both in New York City), or, on the other hand, one may design with just sufficient strength to pass inspection, according to the proposed use and the character of the job. Buildings to be used for sleeping should naturally be steadier than an office building needs to be. Note that amplitude can be reduced by additional bracing, but only with a corresponding increase in speed of vibration. Speed, however, can be reduced without changing the amplitude, by adding weight at the top. The architect's ideal of a "tin tower," made of light weight materials, is, therefore, not consistent with comfort, above a very limited ratio of slenderness.

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[11] "Stiffness of Sky-Scrapers," International Assoc. for Bridge and Structural Eng., Paris, France, May, 1932.

The lines of research sketched herein would require considerable time and money, but if a new skyscraper era were to occur, with higher and more slender towers, such research, continuing the valuable pioneer work reported by Professor Rathbun, seems necessary to obtain a rational method of design for the major factor of commercial success—the occupants' feeling of security.

CLYDE T. MORRIS,[13] M. AM. SOC. C. E. (by letter).[13a]—The fact that so little has been published giving observed data on the stresses and deformations produced in actual buildings by the wind, makes this paper especially valuable, even though definite conclusions and recommendations cannot be drawn from these data.

The observations of the air currents surrounding the building and of the variation of the pressure recorded on the manometers show the futility of extreme refinements in the mathematical calculation of wind stresses. This does not mean, however, that the elastic theory of stress distribution may be discarded, but only that the basic values of the wind pressure used may be in error.

From the observed deflections of the building Professor Rathbun computes the equivalent uniform wind pressure to have been 13.4 lb per sq ft at a wind velocity of about 60 miles per hr (see following Equations (4)). If it is assumed that the pressure increases as the square of the velocity an equivalent pressure of 20 lb per sq ft would be caused by a wind velocity of about 73 miles per hr which would indicate that the usual assumption of 20 lb per sq ft being taken entirely by the steel frame is not too conservative. Conclusion (8), that the masonry increases the rigidity of the building about 350% more than that of the unsupported frame, must not be construed to mean that the masonry increases the load-carrying capacity by that amount.

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[13] Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

[13a] Received by the Secretary October 10, 1938.

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DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

By Leon S. Moisseiff, M. Am. Soc. C. E.

Leon S. Moisseiff,[32] M. Am. Soc. C. E. (by letter).[32a]—The literature on suspension bridges has received an interesting contribution in this paper. A clear understanding of the distortional behavior of suspension bridges, and of the functioning of stiffening trusses, is not as common among engineers as it is generally assumed to be. The facts that, with the exception of suspension bridges, all other bridges may be treated as rigid structures and that suspension bridges are relatively small in number, rightly lead to directing the attention of engineering students to rigid bridges. The distortions of rigid bridges under live load, wind, and temperature are small enough not to affect the primary stresses of their frames. This does not hold true for suspension bridges. In these bridges the distortion of the structure under live load and other outside causes exerts a marked and determining effect on the stresses in the various members of the combined structure. Suspension bridges, therefore, require a different analytical approach than rigid bridges and, as a consequence, the considerations covering their design are also different.

The authors' approach to the problem, through a study of the behavior of an unstiffened cable, is efficient for the purpose of a qualitative appreciation of the functioning of suspension bridges. Such studies were made a century ago by Navier, Rebhan, and others. That these studies dealt with the unstiffened cable was an outcome of the fact that, in those days, suspension bridges were not stiffened systematically. For the light wagon loads of those times a braced railing was considered sufficient. These studies contained much informative material, and offered a good insight into the general behavior of suspension bridges.

Note.—The paper by Shortridge, Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol; June, 1938, by Messrs. Glenn B. Woodruff and Norman C. Raab; September, 1938, by Messrs. Hardy Cross, and A. A. Eremin; and October, 1938, by Messrs. A. Fraser Rose and William A. Rose.

[32] Cons. Engr., New York, N. Y.

[32a] Received by the Secretary October 17, 1938.

After experience had shown that light, unstiffened suspension bridges were neither comfortable for travel nor safe against wind, engineers began to stiffen them. For the short-span bridges built in the early times the attachment of any sizable stiffening girder supplied more than sufficient stiffness. In fact, the girders were so stiff that it appeared to be logical to assume the beam as perfectly rigid. A simple static solution then became possible and was readily established. It formed the basis of the well-known Rankine-Ritter theory. This was the suspension bridge theory taught in engineering schools fifty years ago, which assumed a rigid truss suspended from the cables. The assumption of absolute rigidity of the stiffening truss tacitly involved the unchanged form of the original curve of the cable, and precluded the recognition of the deformations of the structure. The effect of the "dead," or original, load on a suspension bridge could not be considered. The Rankine-Ritter theory ignores it. To this statement should be added that at that period of engineering the elastic behavior of bridges had not as yet come into practical application. In general, bridges were small, relatively stiff, and the elasticity of the members could equally be ignored.

With the greater use of metallic bridges the effect of temperature variation was recognized in the course of time, and a correction was introduced in the stiffening-truss formulas to allow for the change in length of cable due to change in temperature. Soon, thereafter, the elongation of the cables due to change of load also received recognition, and the correction was extended to provide for it.

By that time the study of the elastic behavior of metallic structures had attained a high development and was extended to suspension bridges. The elastic theory, which was then developed, took account of the elasticity of the various parts forming a suspension bridge and forgot the effect of the dead or original load of the bridge. The deflection theory takes this effect into consideration as its basic idea. It has proved that the effect of the dead load or, more precisely, the pull in the cable due to it, is of serious importance and sometimes completely controls the design and behavior of the bridge. The elastic theory has outlived its usefulness as a tool for designing suspension bridges.

The authors are aware of these developments, and they approach the problem with an educated understanding. Step by step, consistently adhering to the function of the stiffening truss and its conditioned behavior in the bridge, the authors approach an intelligent approximation of the more accurate solution. Taking into account the results of the computations of some modern suspension bridges, they make empirical approximations adjusted to these results and arrive at a procedure for a preliminary design.

The comparisons, given in Table 1, of the results obtained by the preliminary method as against those obtained by the deflection theory, are close and satisfactory for the purpose. The range of the data, however, has too narrow a compass.

In the Golden Gate Bridge, in San Francisco, Calif., with a main span of 4 200 ft, dead load of 21 000 lb per lin ft, and live load of 4 000 lb, the greatest positive moment is found at 0.126 of the span length from the tower, and it is

10% greater than at the quarter-point. Attention is also called to the fact that the stress due to live load in the Golden Gate Bridge is only 30% of the total stress and that the remaining 70% is due to the transverse wind.

In the Whitestone Bridge, in New York, N. Y., with a span of 2 300 ft, dead load of 11 000 lb per lin ft, and live load of 3 000 lb, the moment at the quarter-point, by the authors' approximate method, is 5 172 kip-ft, and by the deflection theory, it is 5 000 kip-ft, which is very close; but, again, this moment is not the maximum moment. The maximum moment occurs at one-tenth the span length from the tower, and amounts to 6 360 kip-ft. This is 23% greater than at the quarter-point. Evidently, the assumption that the maximum moment is at the quarter-point in all bridges is incorrect, and the method proposed should be handled with care.

The writer disagrees with the authors that the steps involved at arriving at results by the approximate method will take less time than by using the deflection theory procedure directly. After all, it depends entirely on one's familiarity with this procedure. That a computation machine should be used for rapid and accurate results is true; but an engineering office engaged in designing suspension bridges is supposed to be so equipped.

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DISCUSSIONS

COST OF ENERGY GENERATION

SECOND SYMPOSIUM ON POWER COSTS

Discussion

BY R. H. PARSONS, ESQ.

R. H. PARSONS,[68] ESQ. (by letter).[68a]—Since most of the operating charges of a given hydro-electric system, such as labor, management, maintenance, etc., are almost independent of the load, and as the capital charges are also constant, it is reasonable to assume, as Mr. Page does (see heading, "Effect of Load Factor"), that the cost of producing hydro power will be inversely proportional to the annual load factor. It must be remembered, however, that the consumer of electricity pays only for current at his own terminals, so that the capital charges and operating expenses of long transmission lines usually will have to be added to those of the hydro station before the minimum selling price of the power can be determined.

As far as fixed charges are concerned, of course, the same conditions apply to a steam power station, but the latter has the advantage that it can be built and extended as required by the demand at the time, thus reducing idle capital expenditure to the minimum; and the greater freedom in the choice of its location avoids the cost of long transmission lines. The fixed charges on a steam plant, therefore, are always substantially less than those of a hydro plant, so that as regards these charges only, the cost of steam-generated power to the consumer will be much less affected by a poor load factor on the system. In 1928, Professor A. G. Christie[69] suggested a formula of the type: British thermal unit per kilowatt-hour $= a + \frac{100\,000}{L}$, as showing the effect of the load factor on the heat consumption of a power station. This formula, how-

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NOTE.—This Symposium was presented at the meeting of the Power Division, New York, N. Y., January 20, 1938, and published in April, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1938, by Messrs. Joel D. Justin, Frank H. Mason, W. V. Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, and Edwin D. Dreyfus.

[68] Engr., C. A. Parsons & Co., Ltd., Newcastle-on-Tyne, England.

[68a] Received by the Secretary August 23, 1938.

[69] "Peak Load Problems in Steam Power Stations," by A. G. Christie, read before the meeting of A.S.M.E. at its meeting in New York, N. Y., December, 1928 (F.S.P. 51-24).

ever, is open to the serious objection that it takes no account of the magnitude of the output. Indeed, it would imply that any station running steadily for a year at half load would have the same heat consumption per kilowatt-hour as if it had been running steadily at full load, for the load factor would be the same (100%) in both cases.

Referring to the remarks of Mr. Whitman the point that is often overlooked when comparing the capital costs of hydro and steam plants for a given duty or when considering the part to be played by each in a combined system, is the advantage the steam turbine possesses in respect to overload capacity. By judicious design it is possible to construct a large steam turbine for a maximum output equal to practically double the most economical output, and to attain this result with only a very slight sacrifice of efficiency at the higher loads In power stations, the use of turbines with large overload capacities was fully dealt with in a paper[70] read by Sir C. A. Parsons and R. Dowson at the World Power Conference, at Berlin, Germany, in 1930. It can be shown that turbo-generators having a continuous rating of 75% instead of the more usual 25% in excess of the most economical rating would provide a very large emergency capacity for about one-third the cost of installing extra plant for such emergency, thus giving a lower total cost per unit generated, which after all is more important to the consumer than spectacular thermal efficiency results.

In his recapitulation of the various features of the papers, Mr. Uhl, referring to the depreciation and obsolescence of steam and hydro plants, states that the average rate of depreciation may be as high as 5% for a steam plant and as low as 2% for a hydro plant (see "Elements of Power Cost: (2) Depreciation and Obsolescence"). Taking his own estimates of life, however, it is equally true that the depreciation of a hydro plant may be as high as 2.5%, and that of a steam plant as low as 3 per cent. There are turbo-generators running to-day in British steam stations which were installed 34 yr ago (in 1904), and it is found economical to keep them in service in spite of their obsolete steam conditions and design. Mr. Uhl (see heading "Steam and Water Power Combined") claims advantages for hydro plants over steam plants in the matter of starting-up time. It is certainly true that to start up a steam station with everything cold, would normally take several hours, but such a requirement is so rare that it need scarcely be considered. A turbine, equipped with slow-speed turning gear, can be put on the line from the cold condition in a matter of minutes should this be necessary, and boilers are now being developed which will provide steam from a cold start almost as quickly.

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[70] "The Use in Power Stations of Steam Turbines Having with Their Auxiliaries Large Overload Capacities," by the Hon. Sir Charles A. Parsons and R. Dowson, read at the Second World Power Conference, Berlin, Germany, 1930.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

LATERAL EARTH AND CONCRETE PRESSURES

Discussion

By A. E. Cummings, M. Am. Soc. C. E.

A. E. Cummings,[9] M. Am. Soc. C. E. (by letter).[9a]—An interesting analysis of the problem of lateral pressures is contained in this paper.[9b] However, from Fig. 2 and the supporting remarks, it appears that the authors have misinterpreted the nature of the horizontal stresses due to surface loads.

Referring to Fig. 3(*a*), the complete representation of stress at the point, *O*, would include three normal stresses and three shearing stresses. For the purpose of this discussion, it is not necessary to consider the shearing stresses. The three normal stresses are:

$$p_z = -\frac{3\,P}{2\,\pi}\,\frac{z^3}{\rho^5} \quad \text{(34a)}$$

$$p_y = \frac{P}{2\,\pi}\left[-\frac{3\,y^2\,z}{\rho^5} + (1 - 2\,\mu)\left\{\frac{z}{\rho^3} - \frac{(\rho^2 - y^2)\,(z + \rho) - \rho\,y^2}{\rho^3\,(z + \rho)^2}\right\}\right] \quad \text{(34b)}$$

and,

$$p_x = \frac{P}{2\,\pi}\left[-\frac{3\,x^2\,z}{\rho^5} + (1 - 2\,\mu)\left\{\frac{z}{\rho^3} - \frac{(\rho^2 - x^2)\,(z + \rho) - \rho\,x^2}{\rho^3\,(z + \rho)^2}\right\}\right] \quad \text{(34c)}$$

These stresses are written in rectangular co-ordinates. The plus sign represents tension and the minus sign compression. The subscripts, x, y, and z, refer to the co-ordinate axis with which the stress is parallel. This discussion will concern itself with the stress, p_x, which corresponds to the authors' stress, F, in Equation (12).

Fig. 8 represents the X–Z plane of Fig. 3(*a*). Line $R\,S$ is the trace on the X–Z plane of an imaginary plane which is parallel to the Y–Z plane.

Note.—The paper by Lazarus White and George Paaswell, Members, Am. Soc. C. E., was published in September, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[9] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[9a] Received by the Secretary October 3, 1938.

[9b] Correction for *Transactions:* In the Appendix change the definition of *F* to read "Total lateral pressure per unit of area."

The unit horizontal stress, p_x, at any point on the line, $R\,S$, is given by Equation (34c). Inspection of this equation does not show whether the stress is tension or compression. However, it is a simple matter to determine the nature of this stress at the point, T, which is on the surface of the solid. Substitution of $z = 0$ and $\rho = x$ in Equation (34c) gives:

$$p_x = (1 - 2\,\mu)\,\frac{P}{2\,\pi\,\rho^2} \,. \,. \,(35a)$$

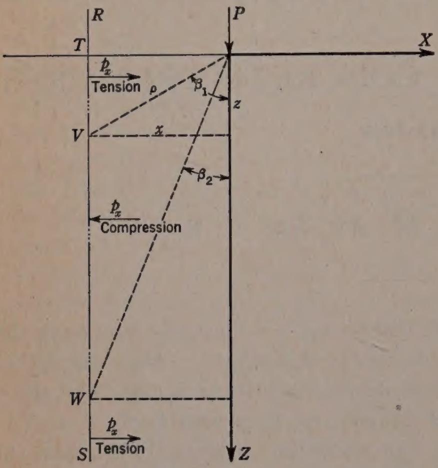

FIG. 8

In a similar manner the nature of this stress at any point on the Z-axis may be determined. Substitution of $x = 0$ and $\rho = z$ in Equation (34c) gives:

$$p_x = (1 - 2\,\mu)\,\frac{P}{4\,\pi\,\rho^2} \,. \,. \,(35b)$$

Since μ is never numerically greater than 0.5 in any elastic solid, the right-hand sides of Equations (35) are positive for all elastic solids. Stress p_x, therefore, is a tension at the point, T, and at all points on the Z-axis. For an incompressible material ($\mu = 0.5$) Stress p_x becomes zero at the point, T, and at all points on the Z-axis. In elastic materials with μ-values less than 0.5, the stress, p_x, on the line, $R\,S$, is a tension from the surface down to some point, V. At the point, V, the stress changes sign and becomes a compression. It is a compression down to the point, W, where it again changes sign and becomes a tension.

The location of the points, V and W, can be determined from Equation (34c) by setting the expression in square brackets equal to zero. In the new equation thus obtained, it is necessary to substitute $x^2 = \rho^2 - z^2$ and $\frac{z}{\rho} = \cos\beta$. Reduction of the equation then leads to,

$$\cos\beta + \cos^2\beta - \cos^3\beta - \cos^4\beta = \frac{1 - 2\,\mu}{3} \,. \,. \,. \,. \,. \,. \,. \,. \,. \,. \,. \,(36)$$

It is seen from Equation (36) that the positions of the points, V and W, are determined by the elastic properties of the material. For a given numerical value of μ, the angle, β, can be found by solving the fourth-order equation in $\cos\beta$. When $\mu = 0.2$, Equation (36) has two real roots which are (approximately) $\cos\beta_1 = 0.175$ and $\cos\beta_2 = 0.945$. The other two roots are a pair of conjugate complex numbers which have no physical meaning in this problem. Therefore, in an elastic solid having $\mu = 0.2$, the angle, β_1, is about 80° and the angle, β_2, is about 19 degrees.

It should be noted that Equation (36) applies only in the X–Z plane. In order to locate the regions of tension and compression throughout the threedimensional space, it is necessary to use Equation (34*c*). The numerical value of μ, together with the numerical values of the co-ordinates of a given point, must be substituted in the equation. If the right-hand side of the equation is positive, the stress is a tension at the given point. When the right-hand side of the equation is negative, the stress is compression. Numerical calculation with Equation (34*c*) is rather laborious although it involves less labor than is required to calculate with Equations (12), (13*a*), and (13*b*).

In a practical problem, the given dimensions would be the rectangular co-ordinates of the point at which the stress was to be determined. These dimensions could be substituted directly in Equation (34*c*). In order to use Equations (12), (13*a*), and (13*b*) it would be necessary to transform the rectangular co-ordinates, x, y, and z, into spherical co-ordinates α, β, and ρ, and then to substitute the spherical co-ordinates in the equations. The writer is unable to understand why the authors chose to handle this rectangular problem in spherical co-ordinates since the method does not simplify the problem in any way.

Furthermore, if it is considered desirable to use angles instead of rectangular co-ordinates, it is much easier to use the direction cosines of the radius vector, ρ, than it is to transform the entire problem to spherical co-ordinates. If β is the angle between ρ and the Z-axis and if ϕ is the angle between ρ and the X-axis, Equation (34*c*) can be expressed in terms of functions of these angles as follows:

$$p_x = \frac{P}{2\,\pi\,\rho^2}\left[-3\cos^2\phi\cos\beta + (1-2\,\mu)\left\{\cos\beta - \frac{\sin^2\phi}{1+\cos\beta}\right\}\right] \quad ..(37)$$

It is easy to see that Equation (37) is much less complicated than Equations (12), (13*a*), and (13*b*).

In connection with Fig. 2, the authors state that they are ignoring any effect that may be caused by the introduction of the free vertical surface. In effect, this means that the wall in Fig. 2 represents nothing more than the imaginary plane, $R\,S$, in Fig. 8. However, the horizontal stress on the line, $R\,S$, is a tension near the surface, and it changes to a compression at some point, V, below the surface. At the point, W, the stress changes again from a compression back to a tension. Even with the simplifying assumption made by the authors, Fig. 2 does not represent the typical distribution of lateral stress on a vertical plane due to a point load on the surface of the semi-infinite elastic solid.

As far as the actual effect of the free vertical surface is concerned, this problem has been studied both theoretically and experimentally. A theoretical analysis has been published by Raymond D. Mindlin,[10] Jun. Am. Soc. C. E. The same problem has been the subject of considerable experimental investigation conducted by M. G. Spangler,[11] Assoc. M. Am. Soc. C. E. Mr. Mindlin's analysis and Mr. Spangler's experiments indicate that the actual pressure on the back of the wall due to a load at the surface is considerably different from Fig. 2. In other words, it has already been demonstrated that the free vertical surface has a decided effect on the stress distribution so that the authors' assumption about this question was not justified.

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[10] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Vol. III, 1936, p. 155.

[11] *Bulletin No. 140*, Iowa Eng. Experiment Station, 1938.

It is interesting to note that the authors' equations themselves, disprove the validity of Fig. 2. Table 1 shows that there are regions in the semi-infinite elastic solid where the horizontal stress, F, is a tension. This table also shows that the value of μ determines the amount of the tension and the region in which the tension occurs. Throughout Table 1, the X–Z plane is represented by $\alpha = 0$. In that part of the table which is calculated for $\mu = 0.2$, the stress for $\alpha = 0$ is tension when β lies between zero and 20° and it is also tension when β lies between 80 and 90 degrees. This agrees exactly with the results obtained from Equation (36).

It can also be seen quite easily that Fig. 2 does not represent the characteristic lateral pressure distribution due to the types of loading shown in Fig. 4. Throughout Fig. 4, the ground surface is represented by $y = 0$. By inspection, it can be seen that the substitution of $y = 0$ in Equation (24) will make $F = 0$. In other words, the stress, F, in Fig. 4(*a*) is zero at the ground surface. Substitution of $y = 0$ in Equations (25*a*), (26*a*), and (27) shows that the stress, F, is zero at the ground surface for all the types of loading shown in Fig. 4.

In conclusion, the writer wishes to call attention to an important point in connection with problems of this kind. Aside from the fact that Fig. 2 does not represent the typical distribution of lateral stress on a vertical plane for any kind of surface loading on a semi-infinite elastic solid, there is the question of ignoring the free vertical surface on the assumption that the neglect of this factor introduces no serious error. Most problems in the theory of elasticity can be described as boundary value problems. The problem is usually set up as a differential equation for which a particular solution is determined by the use of certain boundary conditions. With a different set of boundary conditions, the solution of the differential equation may be entirely different. For all the various types of surface loads mentioned in this paper, the authors' equations for F were derived on the assumption that the elastic solid was infinitely deep and that its plane horizontal boundary extended to infinity in both horizontal directions. The solutions of the differential equations were required to satisfy certain conditions at infinity and certain other conditions on the free horizontal boundary. Equations derived for these boundary conditions could not be expected to be valid when a new boundary condition is introduced in the form of a free vertical surface not far from the load. Mr. Spangler's experiments[11] show that the maximum horizontal pressures on the back of the wall could be at least twice as great as those calculated with Equation (12) even in a material having $\mu = 0.5$. For smaller values of μ, Equation (12) gives tensions in certain parts of the stress field but tension could not exist unless the soil were somehow fastened to the back of the wall. These are conditions that should be kept in mind by any one attempting to calculate pressures on retaining walls with the equations in this paper.